Commun. Comput. Phys.
doi: 10.4208/cicp.160115.150915a

Vol. **18**, No. 5, pp. 1264-1281
November 2015

High Order Numerical Simulation of Detonation Wave Propagation Through Complex Obstacles with the Inverse Lax-Wendroff Treatment

Cheng Wang[1,*], Jianxu Ding[1], Sirui Tan[2] and Wenhu Han[3]

[1] *State Key Laboratory of Explosion Science and Technology, Beijing Institute of Technology, Beijing, 100081, P.R. China.*
[2] *Division of Applied Mathematics, Brown University, Providence, RI 02912, USA.*
[3] *Center for Combustion Energy, Tsinghua University, Beijing, 100084, P.R. China.*

Communicated by Chi-Wang Shu

Received 16 January 2015; Accepted (in revised version) 15 September 2015

Abstract. The high order inverse Lax-Wendroff (ILW) procedure is extended to boundary treatment involving complex geometries on a Cartesian mesh. Our method ensures that the numerical resolution at the vicinity of the boundary and the inner domain keeps the fifth order accuracy for the system of the reactive Euler equations with the two-step reaction model. Shock wave propagation in a tube with an array of rectangular grooves is first numerically simulated by combining a fifth order weighted essentially non-oscillatory (WENO) scheme and the ILW boundary treatment. Compared with the experimental results, the ILW treatment accurately captures the evolution of shock wave during the interactions of the shock waves with the complex obstacles. Excellent agreement between our numerical results and the experimental ones further demonstrates the reliability and accuracy of the ILW treatment. Compared with the immersed boundary method (IBM), it is clear that the influence on pressure peaks in the reflected zone is obviously bigger than that in the diffracted zone. Furthermore, we also simulate the propagation process of detonation wave in a tube with three different widths of wall-mounted rectangular obstacles located on the lower wall. It is shown that the shock pressure along a horizontal line near the rectangular obstacles gradually decreases, and the detonation cellular size become large and irregular with the decrease of the obstacle width.

AMS subject classifications: 65M06, 76L05

Key words: Boundary treatment, detonation, inverse Lax-Wendroff, two-step reaction model, WENO scheme.

---

*Corresponding author. *Email address:* `wangcheng@bit.edu.cn` (C. Wang)

1 Introduction

Studies of the propagating detonation wave in a tube with complex boundaries are an important research field for not only fundamental sciences, but also engineering applications, which usually requires numerical simulations be carried out for the intricate geometries intersecting the Cartesian meshes in an arbitrary fashion. Even though a high order numerical scheme in the internal computational domain is adopted, most of boundary treatments based on the Cartesian grids are not more than the second order accuracy in the vicinity of the complex boundaries. The numerical error caused by a traditional and low order boundary treatment may pollute the results of the internal computational domain. Recently, some successful boundary treatment techniques based on the Cartesian grids are proposed and developed. The reflection technique [1] is a very simple and popular boundary treatment method, in which all physical variables are reflected symmetrically on the rigid wall except for the anti-symmetric normal velocity. This method works well when the rigid boundary is straight and positioned at half points. However, large errors might be caused when such method is applied to complex boundary [2]. Later, the IBM first proposed by Peskin [3] has been numerously modified and refined to deal with both incompressible flows and compressible flows including strong shock waves. Sjögreen et al. [4] developed an embedded boundary finite difference technique with the second order accuracy for solving multi-dimensional Euler equations. To avoid numerical oscillations near shock waves, a slope limiter was adopted on complex boundary. Chaudhuri et al. [5] described the implementation of IBM and then applied it in conjunction with a fifth order classical WENO scheme to simulate the complex fluid-solid interactions. The simplified and modified ghost point treatments were introduced by Farooq et al. [6] for solving two dimensional compressible Euler equations based on Cartesian grids. Their methods were second order accurate near boundaries. Tan and Shu [7] developed a high order numerical boundary treatment, consisting of the inverse Lax-Wendroff type procedure for inflow boundary conditions and extrapolation for outflow boundary conditions, to solve hyperbolic conservation laws with finite difference methods. However, the algebra of the ILW procedure depended on partial difference equations (PDEs) was very heavy for practical applications. In [8], the boundary treatment proposed by Tan et al. [7] was further simplified and improved for efficient implementation in practice. This innovative boundary treatment procedure was demonstrated on the scheme with fifth order accuracy which was the same order as the internal WENO scheme. Vilar and Shu [9] generalized the ILW treatment for the inflow boundary of a linear hyperbolic problem and analyzed the stability of the resulting schemes.

The mechanism of detonation wave propagation in complex tubes was studied widely by many researchers, which is of great importance to industrial safety. Up to now, some experimental and numerical results are available. The experiment on gaseous detonation diffraction in a 90 degrees branched channel was investigated by the work of Guo et al. [10]. Starr et al. [11] performed studies on detonation limits in rough tubes. It was concluded that detonation limits in rough tubes were wider than that in smooth tubes.