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Pseudo-Depth/Intercept-Time Monotonicity Requirements in the Inverse Scattering Algorithm for Predicting Internal Multiple Reflections

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Abstract. In this paper we discuss the inverse scattering algorithm for predicting internal multiple reflections (reverberation artefacts), focusing our attention on the construction mechanisms. Roughly speaking, the algorithm combines amplitude and phase information of three different arrivals (sub-events) in the data set to predict one multiple reflection. The three events are conditioned by a certain relation which requires that their pseudo-depths, defined as the depths of their turning points relative to the constant background velocity, satisfy a lower-higher-lower relationship. This implicitly assumes a pseudo-depth monotonicity condition, i.e., the relation between the actual depths and the pseudo-depths of any two sub-events is the same. We study this relation in pseudo-depth and show that it is directly connected with a similar relation between the vertical or intercept times of the sub-events. The paper also provides the first multidimensional analysis of the algorithm (for a vertically varying acoustic model) with analytical data. We show that the construction of internal multiples is performed in the plane waves domain and, as a consequence, the internal multiples with headwaves sub-events are also predicted by the algorithm. Furthermore we analyze the differences between the time monotonicity condition in vertical or intercept time and total travel time and show a 2D example which satisfies the former but not the latter. Finally we discuss one case in which the monotonicity condition is not satisfied by the sub-events of an internal multiple and discuss ways of lowering these restrictions and of expanding the algorithm to address these types of multiples.

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1 Introduction

The inverse scattering series is presently the only multidimensional method for inverting for the properties of an unknown medium without adequate information about that medium. When the series converges it achieves full inversion given the whole data set (including free surface reverberations and internal multiple reflections) and information about a chosen reference medium. Carvalho [6] tested numerically the convergence properties of the full inverse scattering series and found that the series converges only for limited contrast between the actual and the reference medium of choice. In the '90's, Weglein and collaborators developed the subseries method (for a history and description see [18]) which consists in identifying specific subseries in the full series, which perform targeted tasks with better convergence properties than the whole series. These subseries were imagined to be a sequence of steps, similar to the processing steps undertaken in geophysical exploration, which would achieve

1. Free surface multiple elimination;
2. Internal multiple elimination;
3. Imaging in depth;
4. Inversion for the medium properties.

It is reasonable to assume, and experience showed this assumption to be true, that since the full series only requires data and information about a reference medium to invert, the same holds for any of the four specific subseries.

The inverse scattering series, and the subsequent task specific subseries, assume that the input data satisfies several pre-requisites. First, it is assumed that the source signature or wavelet has been deconvolved from the data. Second, both the source and receiver ghosts (the part of the wavefield which travels from the source to the free surface and from the free surface to the receiver) have been eliminated from the collected data. Third, the collected data itself has an appropriate sampling or the data reconstruction algorithms are able to improve the acquisition sampling to an appropriate degree. When these prerequisites are not satisfied, the algorithms derived from this method will reach incorrect conclusions/results, e.g., false or no prediction of free-surface and internal multiples, incorrect location of subsurface structures, errors in parameter estimation. Last but not least we mention that the algorithms are derived from a point-source point-receiver wave theory approach and any deviations from that, e.g., source and receiver arrays, would have to be studied to understand how they affect the algorithms.

In 1994, Araujo [2] identified the first term in the subseries for internal multiple elimination (see also [17]). This first term by itself exactly predicts the time of arrival, or phase, and well approximates the amplitude of internal multiples, without being larger than the actual amplitude, and hence it represents an algorithm for *attenuation*. Weglein et al. [18] described the application of the algorithm to 1D analytic and 2D synthetic data. Field data tests were also performed showing an extraordinary ability to predict difficult