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# The Simplified Lattice Boltzmann Method on Non-Uniform Meshes

Zhen Chen, Chang Shu* and Danielle S. Tan

*Department of Mechanical Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore.*

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**Abstract.** In this paper, we present an improved version of the simplified lattice Boltzmann method on non-uniform meshes. This method is based on the recently-proposed simplified lattice Boltzmann method (SLBM) without evolution of the distribution functions. In SLBM, the macroscopic variables, rather than the distribution functions, are directly updated. Therefore, SLBM calls for lower cost in virtual memories and can directly implement physical boundary conditions. However, one big issue in SLBM is the lattice uniformity, which is inherited from the standard LBM and this makes SLBM only applicable on uniform meshes. To further extend SLBM to non-uniform meshes, Lagrange interpolation algorithm is introduced in this paper to determine quantities at positions where the streaming process is initiated. The theoretical foundation of the interpolation process is that both the equilibrium part and the non-equilibrium part of the distribution functions are continuous in physical space. In practical implementation, the Lagrange interpolated polynomials can be computed and stored in advance, due to which little extra efforts are brought into the computation. Three numerical tests are conducted with good agreements to reference data, which validates the robustness of the present method and shows its potential for applications to non-uniform meshes with curved boundaries.

**AMS subject classifications**: 76M28

**Key words**: Simplified lattice Boltzmann method, lattice uniformity, Lagrange interpolation, non-uniform meshes, distribution functions.

## 1 Introduction

The lattice Boltzmann method (LBM), known as a mesoscopic approach to solve fluid mechanics problems, attracted increasing attentions in recent decades [1–4]. Due to its kinetic nature, simplicity and explicit formulations, LBM has been broadly applied in various problems [5–12]. However, standard LBM is, to some extent, constrained by several

*Corresponding author. *Email addresses:* `chenzhen@u.nus.edu` (Z. Chen), `mpeshuc@nus.edu.sg` (C. Shu), `mpe_dtan@nus.edu.sg` (D. S. Tan)

 

drawbacks, including high request on virtual memories, inconvenient implementation of physical boundary conditions, the numerical instability and the lattice uniformity.

To alleviate the drawbacks of standard LBM, the simplified lattice Boltzmann method (SLBM) was recently developed [13]. With Chapman-Enskog (C-E) expansion analysis, the lattice Boltzmann equation (LBE) can recover the macroscopic Navier-Stokes (N-S) equations. With the assistance of fractional step technique, the recovered macroscopic equations could be resolved in a predictor-corrector scheme. In SLBM, the formulations in the predictor-corrector steps are reconstructed within LBM frame, by using the lattice properties and the relationships given from C-E analysis; and the resultant formulations in SLBM only involve the equilibrium and the non-equilibrium distribution functions. In practical implementations, the equilibrium distribution function is calculated from the macroscopic variables, while the non-equilibrium distribution function is simply evaluated from the difference of two equilibrium distribution functions. As a result, SLBM directly tracks the evolution of the macroscopic flow variables rather than the distribution function, which gives it many attracting characteristics. On one hand, compared to the standard LBM, the simplified LBM requires lower virtual memories and facilitates the implementation of physical boundary conditions. On the other hand, since SLBM is developed within LBM framework, the advantages of the standard LBM are maintained. However, SLBM also inherits one drawback of the standard LBM: lattice uniformity. The lattice uniformity comes from the symmetric lattice velocity model applied in the lattice Boltzmann method. Due to the lattice uniformity, the fluid particles must be streamed from uniformly distributed surrounding points to ensure that the local collision process can happen at the central grid point. Such characteristic makes the simplified lattice Boltzmann method only applicable on uniform meshes, which cannot be directly applied to practical problems where the non-uniform meshes are usually preferred to accurately describe the curved boundaries and to achieve higher computational efficiency. Such conflict gives us the motivation to extend the simplified lattice Boltzmann method to non-uniform meshes.

The ideas applied in previous studies to extend standard LBM to non-uniform meshes are of reference value to the present research. Basically, to the best of our knowledge, three approaches have been proposed by various researchers to apply the standard LBM on non-uniform meshes. One is the interpolation-supplemented LBM (IS-LBM) [14, 15]. Another way is to combine the differential lattice Boltzmann equation with finite difference method (FDLBE) and coordinate transformation [16], or with finite volume method (FVLBE) [17–19]. The third way is the recently developed Taylor-series expansion and least-squares-based lattice Boltzmann method (TLLBM) [20,21]. It is noted that in FDLBE or FVLBE methods, special techniques, such as upwind schemes, are introduced to stabilize the computation. In IS-LBM and TLLBM, the basic idea is to apply the interpolation or the fitting algorithms into the standard LBM. Such treatment is based on a reasonable assumption that the distribution function is continuous in physical space and is more straightforward in derivations and implementations.

Inspired by the ideas presented in IS-LBM, we propose an improved version of sim-