Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0011

Vol. **22**, No. 4, pp. 1101-1122
October 2017

Electrophoresis of a Cylinder in a Cylindrical Tube

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Received 17 January 2017; Accepted (in revised version) 24 March 2017

Abstract. Electrophoresis of a cylinder suspended in a cylindrical tube is analytically studied in the limit of thin electric double layer approximation. The electric and fluid flow fields within the annulus, and the cylinder velocities are analytically obtained in bipolar coordinates. The results are analyzed with various values of dimensionless parameters: eccentricity, cylinder-to-tube radius ratio and tube-to-cylinder zeta potential ratio (i.e., tube-to-cylinder velocity scale ratio). The analysis shows that microvortices are generated within the annulus. By changing the parameters, different flow patterns can be created, which shows potential for mixing enhancement in micro/nanofluidics. Moreover, the cylinder not only translates but also rotates when the cylinder and tube are eccentric. The cylinder rotation might be utilized as a micromotor or an electric field detector. The cylinder trajectories show that the cylinder may approach the tube wall or rest within the tube depending on the zeta potential ratio.

AMS subject classifications: 76W05

Key words: Electrophoresis, cylindrical tube, vortex generation, cylinder rotation.

1 Introduction

Electrokinetics has been widely studied due to its potential applications in various areas, ranging from micropumps [2,3], micromixers [4,5], microvalves [6,7], to particle trapping [8]. When a charged particle is suspended in an electrolyte solution, counterions are attracted to the particle surface, forming an electric double layer (EDL). After an external electric field is applied, an electrical force is exerted on the ions within the EDL. The ions are driven into motion, which leads to a bulk fluid flow, known as electroosmosis. In the limits of thin EDL (relative to the local radii of curvature of the particle) and small zeta potential ($\zeta < 25$ mV) approximations, the ion motion within the EDL acts as a slip

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velocity on the particle surface, described by the well-known Helmholtz-Smoluchowski formula,

$$\mathbf{u}_s = -\frac{\varepsilon_w \zeta}{\mu}\mathbf{E}_t, \tag{1.1}$$

where ε_w and μ are the dielectric permittivity and the viscosity of the electrolyte solution, respectively; ζ is the zeta potential; $\mathbf{E}_t$ is the tangential component of the electric field on the particle surface.

Corresponding to the electroosmotic flow, the particle is driven towards the oppositely charged electrode, known as electrophoresis. Electrophoresis has long been utilized for bubble [9], droplet [10], and particle [11, 12] manipulations in various chemical and biomedical applications. The electrophoretic velocity $\mathbf{U}$ of a particle of arbitrary shape and thin double layer in an unbounded electrolyte solution is related to the uniformly applied electric field $\mathbf{E}_\infty$ through the equation $\mathbf{U} = \varepsilon_w \zeta \mathbf{E}_\infty / \mu$. As the EDL thickness is generally in the order of nanometers or tens of nanometers, which is much smaller than the typical size of commonly encountered particles, this equation has been widely used. However, in microfluidic and other practical applications, the particles are not isolated and move near boundaries. Therefore, it is of great importance to investigate boundary effect in electrophoresis. The boundary effects on the electrophoresis of a spherical particle have been widely studied for various cases, e.g., migration normal to a conducting planar wall [13], along a dielectric planar wall [14], within a spherical cavity [15], in a cylindrical tube [16, 17], and between two planar walls [18, 19]. The electrophoresis of a soft spherical particle within a spherical cavity [20] and a cylindrical tube [21] have been reported. The electrophoresis of a soft toroid has been characterized along the axis of a cylindrical tube [22]. Studies on soft particles could contribute to the understanding of the electrophoresis of biocells. The analytical solution of the electrophoresis of a cylinder near a planar wall has been obtained in bipolar coordinates [23]. The electrophoretic interactions between a conducting and a non-conducting cylinder have been analytically characterized [24]. Surprisingly, cylinder rotation was revealed in the pair interactions [24]. The electrophoresis of a finite cylindrical particle has been analyzed along the axis of a cylindrical tube [25]. Studies have also been carried out on the electrophoresis of particles in porus media [26, 27] and in non-Newtonian fluids [28, 29].

However, it was not clear how a non-conducting cylinder inside a cylindrical tube behaves. In an effort to improve the physical insight into this problem, we hereby carry out a comprehensive study on the electrophoresis of a non-conducting cylinder suspended in a non-conducting cylindrical tube. The fluid flow field and the cylinder velocities are analytically obtained in bipolar coordinates, and examined in detail. The results show that microvortices are generated within the annulus, which shows potential for mixing enhancement in micro/nanofluidics. Furthermore, the cylinder not only translates but also, surprisingly, rotates, when it is eccentric with respect to the tube. Such cylinder rotation may be utilized as a micromotor or an electric field detector. It has advantages of simple geometry and material property compared to the structures composed of different materials [30, 31].