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A Two-Stage Image Segmentation Model for Multi-Channel Images

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Abstract. This paper introduces a two-stage model for multi-channel image segmentation, which is motivated by minimal surface theory. Indeed, in the first stage, we acquire a smooth solution u from a convex variational model related to minimal surface property and different data fidelity terms are considered. This minimization problem is solved efficiently by the classical primal-dual approach. In the second stage, we adopt thresholding to segment the smoothed image u. Here, instead of using K-means to determine the thresholds, we propose a more stable hill-climbing procedure to locate the peaks on the 3D histogram of u as thresholds, in the meantime, this algorithm can also detect the number of segments. Finally, numerical results demonstrate that the proposed method is very robust against noise and superior to other image segmentation approaches.

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Key words: Image segmentation, minimal surface, multi-channel, primal-dual method, total variation.

1 Introduction

Image segmentation is a fundamental and challenging topic in computer vision and image processing. For instance, dividing the digital photo into reasonable parts is the first task for gaining a meaningful understanding of the image, and sometimes more than one segmentation result are required. Over the past few decades, we have seen an explosive growth in the diversity of image segmentation techniques, such as PDE-based approaches [20, 37, 54], and thresholding methods [40, 45]. In this paper, we attempt to eliminate the weakness in the classic PDE models [37], and present an interdisciplinary framework to utilize the strengths from both the PDE-based method and thresholding.

To begin with, we briefly introduce the Mumford-Shah (MS) model [37] and the Potts model [16] to locate the bottlenecks. Typical segmentation models based on those two

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models are analyzed to further specify the drawbacks. As a classical variational problem, the energy of the MS model can be written as

$$E_{MS}(u,\Gamma)=\mu\int_{\Omega}(u-f)^2dx+\int_{\Omega-\Gamma}|\nabla u|^2dx+\lambda|\Gamma|, \tag{1.1}$$

where u is the approximation of the image f, Γ is the collection of boundaries of the partition Ω_i, $\cup_{i=1}^{K}\Omega_i\cup\Gamma=\Omega$, $\Omega_i\cap\Omega_j=\emptyset$, $\forall i\neq j$. Here $|\Gamma|$ denotes the summation of the perimeter of each segment Ω_i, and μ, λ are positive parameters.

The MS model (1.1) has two unknowns of different types: the piecewise smooth image u and the boundary Γ. With small variation on each Ω_i, u is differentiable on the open sets Ω_i, and u can be discontinuous on the boundaries Γ. The model prefers tight curves Γ separating u.

A straightforward simplification of the MS model is restricting $u=c_i$ on each Ω_i (c_i is a constant, $i=1,\cdots,K$, K is the number of partitions Ω_i), the resulting energy of the piecewise constant MS model [20,37] is as follow

$$E(c_i,\Gamma)=\sum_{i=1}^{K}\int_{\Omega_i}(c_i-f)^2dx+\lambda|\Gamma|, \tag{1.2}$$

and this model is also known as the cartoon limit [12].

This simplified model (1.2) is still hard to solve even one of the 'tough term' has been removed, because it inherits the two difficulties from the MS model (1.1): (i) the non-convexity of the problem, (ii) two different types of unknowns (a function u and a contour Γ) in the model.

Another widely studied problem is the Potts model [42] which is a generalization of the Ising model [28]. In the image segmentation, the spatially continuous setting of the Potts problem is of the form [16,54]

$$\min_{\{\Omega_i\}_{i=1}^{k}}\sum_{i=1}^{K}\int_{\Omega_i}g_i(x)dx+\lambda|\Gamma|, \tag{1.3}$$

where g_i measures the costs of assigning the respective pixel to Ω_i, the rest of notations have the same definition as in (1.1) and (1.2). This model attempts to locate K disjoint subdomain Ω_i with tight boundaries dividing continuous domain Ω, and it is also referred to as the minimal partition problem [16].

The Potts model has close connection with the MS model, when $g_i=(c_i-f)^2$, it is equivalent to the piecewise constant MS model.

Moreover, the MS model can be regarded as an approximation of the Potts model. Indeed, if $g_i=-\log p_i(f|x)$, the Potts model becomes the Bayesian model (formula (9) in [12]). More specifically, p_i follows the Gaussian distribution [43]:

$$p_i(f|x)=\frac{1}{\sqrt{2\pi}\sigma_i(x)}\exp\left(-\frac{(f-\mu_i(x))^2}{2\sigma_i(x)^2}\right),$$