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Computational Modeling of Membrane Viscosity of Red Blood Cells

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Abstract. Despite its demonstrated importance in the deformation and dynamics of red blood cells, membrane viscosity has not received the same attention in computational models as elasticity and bending stiffness. Recent experiments on red blood cells indicated a power law response due to membrane viscosity. This is potentially much different from the solid viscoelastic models, such as Kelvin-Voigt and standard linear solid (SLS), currently used in computation to describe this aspect of the membrane. Within the context of a framework based on lattice Boltzmann and immersed boundary methods, we introduce SLS and power law models for membrane viscosity. We compare how the Kelvin-Voigt (as approximated by SLS) and power law models alter the deformation and dynamics of a spherical capsule in shear flows.

AMS subject classifications: 74F10

Key words: Capsule, membrane viscosity, viscoelastic models, power law models.

1 Intro

Recent years have seen the deformation and dynamics of red blood cells studied computationally, as a fluid-structure interaction problem involving a fluid-filled capsule (e.g., [13, 21, 23, 28, 36]). Significant progress has been made in increasing the fidelity of the capsule model, particularly with respect to the properties of red blood cell membranes. The red blood cell membrane has two principal components: the cytoskeleton and lipid bilayer. The cytoskeleton has the property of shear elasticity, the bilayer has bending stiffness and viscosity, and the surface incompressibility of both components leads to isotropic elasticity [3]. Shear and isotropic elasticity, along with bending stiffness, have been well-integrated into two and three dimensional models. The Skalak constitutive

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law has proven effective in accounting for both elastic behaviours [4], while Helfrich's bending energy formulation models biomechanical bending resistance [27].

A clear accounting for the bilayer's viscosity, however, remains somewhat elusive. The pairing of the elastic skeleton and viscous bilayer, subject to the same strain, naturally suggests the Kelvin-Voigt viscoelastic model [19]. The Kelvin-Voigt model has the additional advantage of simplicity, as Evans and Hochmuth employed it to derive an analytical model for stretch recovery from micropipette aspiration [14]. Many more recent theoretical and computational models have also used Kelvin-Voigt (e.g., [5, 9, 40]). Yazdani and Bagchi noted numerical instabilities when implementing Kelvin-Voigt in 3D, opting instead for the more versatile standard linear solid (SLS) model and adjusting parameters so as to approximate Kelvin-Voigt [35]. Studies using stochastic mesoscopic methods have opted for other viscous models. For instance, Fedosov *et al.* developed a general dissipative model within the framework of dissipative particle dynamics [15], while Noguchi and Gompper's multi-particle collision dynamics simulations of vesicles used bond-flipping for membrane viscosity [22].

The viscoelasticity of red blood cells was studied experimentally by Puig-de-Morales-Marinkovic *et al.*, using optical magnetic twisting cytometry (MTC) [29]. They found that the dynamic response of RBCs could not be explained by the prevailing solid viscoelastic models. Instead, their data fit well with a power law, like the models used for non-Newtonian fluids. Since then, power law responses have been noted by experimentalists using optical tweezing [37, 38], dynamic scattering microscopy [1], and diffraction phase microscopy [34]. Fedosov *et al.* simulated twisted torque cytometry [15] and their results agree well with [29], though with a slightly larger power law exponent.

Accordingly, we introduce a pair of models for the cell's membrane viscosity, using standard linear solid and power law frameworks. These are included in a larger simulation methodology, using a lattice Boltzmann method for the fluids and a finite element model for the capsule, and handling the fluid-structure interaction with an immersed boundary method. We consider the deformation and dynamics of a spherical capsule in shear flow and compare the two approaches to the membrane viscosity.

2 General algorithms

2.1 Fluid

Rather than a traditional method of solving the incompressible Navier-Stokes equations, we employ a lattice Boltzmann method (LBM). As opposed to the macroscopic Navier-Stokes equations, for which the variables are fluid velocities and pressure, the mesoscopic LBM simulates the fluid using probability distribution functions. These probability distribution functions represent the averaged behavior of sets of particles, which move between lattice nodes in discrete timesteps with discrete velocities: $\mathbf{f}_i(\mathbf{x}_j, t_n)$ stands for the particle distribution at $\mathbf{x}_j$ with velocity $\mathbf{c}_i$ at time t_n. Based on this variable, we have the