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Constrained Interpolation Profile Conservative Semi-Lagrangian Scheme Based on Third-Order Polynomial Functions and Essentially Non-Oscillatory (CIP-CSL3ENO) Scheme

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Abstract. We propose a fully conservative and less oscillatory multi-moment scheme for the approximation of hyperbolic conservation laws. The proposed scheme (CIP-CSL3ENO) is based on two CIP-CSL3 schemes and the essentially non-oscillatory (ENO) scheme. In this paper, we also propose an ENO indicator for the multi-moment framework, which intentionally selects non-smooth stencil but can efficiently minimize numerical oscillations. The proposed scheme is validated through various benchmark problems and a comparison with an experiment of two droplets collision/separation. The CIP-CSL3ENO scheme shows approximately fourth-order accuracy for smooth solution, and captures discontinuities and smooth solutions simultaneously without numerical oscillations for solutions which include discontinuities. The numerical results of two droplets collision/separation (3D free surface flow simulation) show that the CIP-CSL3ENO scheme can be applied to various types of fluid problems not only compressible flow problems but also incompressible and 3D free surface flow problems.

AMS subject classifications: 35L65, 65M06, 65M08, 76T10

Key words: CIP-CSL scheme, ENO scheme, conservation laws, free surface flows.

1 Introduction

In this paper, we propose a less oscillatory multi-moment method for the approximation of hyperbolic conservation laws

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$$\frac{\partial\phi}{\partial t}+\frac{\partial(u\phi)}{\partial x}=0, \tag{1.1}$$

where ϕ is the scalar and u is the velocity. The proposed method is based on the constrained interpolation profile-conservative semi-Lagrangian (CIP-CSL) scheme [12,21,25, 27] and the essentially non-oscillatory (ENO) scheme [3,16–18].

The CIP-CSL scheme is a solver of conservation laws based on a multi-moment concept which uses cell average and boundary value as moments (variables), and has been applied to various types of fluid problems [22–24] including interfacial flows such as droplet splashing [29, 30]. There are several variants of the CIP-CSL scheme such as CIP-CSL2 (CIP-CSL with second-order polynomial function) [27] and CIP-CSL3 (CIP-CSL with third-order polynomial function) [25]. In CSL2 which is based on a second-order polynomial interpolation function, three moments within the upwind cell (i.e. a cell average and two boundary values) are used for the interpolation function as shown in Fig. 1 (a). In CSL3 which is based on a third-order polynomial interpolation function, a derivative at cell center is used as an additional constraint as shown in Fig. 1 (b). The derivative is a control parameter and mainly used as a limiter. Recently the CIP-CSL3D scheme (D of CSL3D stands for Downwind) was proposed [12]. Although CSL3D also uses a third-order polynomial interpolation function like CSL3, a moment in the downwind cell is used as the additional constraint instead of the derivative in CSL3 (Fig. 1 (c)). In this paper, we propose another variant of CSL3, which will be called CSL3U and which uses an additional moment in upwind side instead of a moment in the downwind cell of CIP-CSL3D (U of CSL3U stands for Upwind) as shown in Fig. 1 (d). By combining CSL3U and CSL3D using the idea of the ENO scheme, we develop a less oscillatory CSL3 (hereinafter referred to as CIP-CSL3ENO scheme). The CIP-CSL3ENO scheme maintains approximately fourth-order accuracy unlike CSL3 schemes which use a standard limiter.

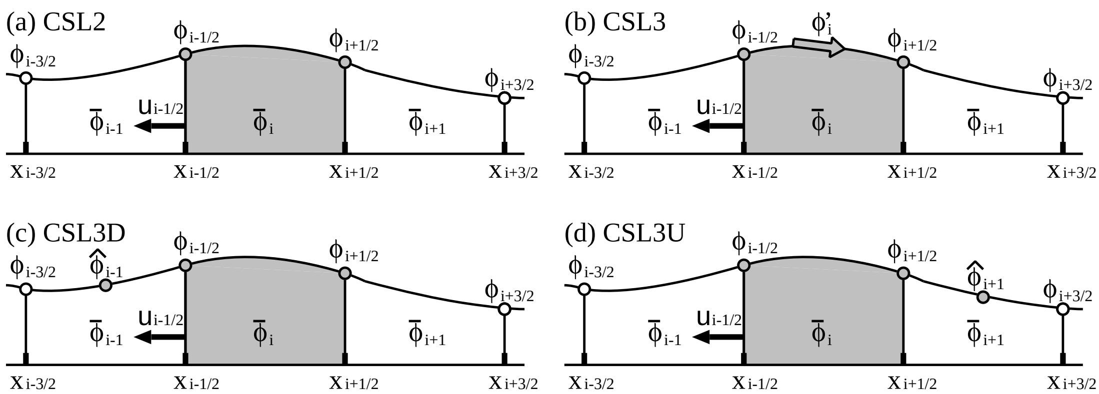

Figure 1: Schematic figures of the CIP-CSL2 method (a), the CIP-CSL3 method (b), the CIP-CSL3D method (c) and the CIP-CL3U method (d). $u_{i-1/2}<0$ is assumed. The moments which are indicated by gray color are used as constraints to construct interpolation functions $\Phi_i^{CSL2}(x)$, $\Phi_i^{CSL3}(x)$, $\Phi_i^{CSL3D}(x)$ and $\Phi_i^{CSL3U}(x)$.