

## Modification and Numerical Method for the Jiles-Atherton Hysteresis Model

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**Abstract.** The Jiles-Atherton (J-A) model is a commonly used physics-based model in describing the hysteresis characteristics of ferromagnetic materials. However, citations and interpretation of this model in literature have been non-uniform. Solution methods for solving numerically this model has not been studied adequately. In this paper, through analyzing the mathematical properties of equations and the physical mechanism of energy conservation, we point out some unreasonable descriptions of this model and develop a relatively more accurate, modified J-A model together with its numerical solution method. Our method employs a fixed point method to compute anhysteretic magnetization. We obtain the susceptibility value of the anhysteretic magnetization analytically and apply the 4th order Runge-Kutta method to the solution of total magnetization. Computational errors are estimated and then precisions of the solving method in describing various materials are verified. At last, through analyzing the effects of the accelerating method, iterative error and step size on the computational errors, we optimize the numerical method to achieve the effects of high precision and short computing time. From analysis, we determine the range of best values of some key parameters for fast and accurate computation.

**AMS subject classifications:** 65L06, 34C55, 39B12, 82D40

**Key words:** Modified J-A model, numerical solution method, error estimation, optimization.

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## 1 Introduction

As a type of functional material, the giant magnetostrictive material (GMM) can generate a field-induced deformation under external magnetic field. With the help of some necessary devices, the material can output required displacement and force [1]. The material

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has many advantages just as high output force, quick responding speed and high Curie temperature, etc. So its application has become a research hotspot [2, 3]. As a kind of ferromagnetic material, giant magnetostrictive material has the hysteresis characteristics. That is to say, the magnetization of the material changes after the magnetic field changing in the time domain. And a residual magnetization exists even the external magnetic field is changed to 0.

Micro-magnetism theory has been the most recognized method to explain the formation of magnetic hysteresis. According to the theory, magnetic domains in different shapes and sizes exist in the ferromagnetic material. The connections and interactions between these domains lead to irreversible movements of the domains. The irreversible movements cause energy losses in the process of forming magnetization, and then the magnetic hysteresis exists. In describing the hysteresis, there were two kinds of models, respectively named as physics-based and phenomenon-based models. The physics-based models considered the micro-mechanism of hysteresis process, including Stoner-Wolhfath model [4], Jiles-Atherton model [5, 6] and Smith free-energy model [7]. The phenomenon-based models, such as Preisach model [8] and neural network model [9], just considered the external behavior of the magnetization caused by the external magnetic field while not the internal mechanism.

As a commonly used hysteresis model, the Jiles-Atherton (J-A) model has been modified for many versions. These versions have different mathematical expressions in describing the magnetization. The inconsistencies are mainly in three aspects, respectively of the format of anhysteretic magnetization [10,11], energy conservation equation [12–14] and the necessary of introduced parameter  $\delta_M$  [15, 16]. These uncertain citations have made these modified J-A models quite difficult to recognize and are really inconvenient for further modeling. Though almost all expressions have been proved in different references [17–20], we notice that some inaccuracies in their manuscripts could be recognized directly from physical or mathematical points of view.

Numerical methods for solving the J-A model also require careful examination. With nonlinear forms, the J-A model cannot be solved analytically. Therefore, the numerical solution is quite important for precise model or control in applying the J-A hysteresis model. However, the solution process and calculation error have rarely been studied [21,22]. In fact, solving the J-A model involves solving nonlinear algebraic equations, nonlinear differential equations and the derivative of nonlinear function, some numerical methods just like interpolation, iteration or fitting methods should be employed. If the calculating method were set unreasonably, the error of the calculated would not be acceptably low.

So the accurate modification and effective solution method, combined with the error analysis, for the J-A hysteresis model are really needed. In this paper, three uninformed points in various J-A model expressions are analyzed. Then a modified J-A model, which could accord to the physical mechanism and mathematical properties of the equations at the same time, is presented and verified. Referring to the form of modified J-A model, the fixed point method and 4<sup>th</sup> order Runge-Kutta method are employed in the numer-