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Effect of Geometric Conservation Law on Improving Spatial Accuracy for Finite Difference Schemes on Two-Dimensional Nonsmooth Grids

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Abstract. It is well known that grid discontinuities have significant impact on the performance of finite difference schemes (FDSs). The geometric conservation law (GCL) is very important for FDSs on reducing numerical oscillations and ensuring free-stream preservation in curvilinear coordinate system. It is not quite clear how GCL works in finite difference method and how GCL errors affect spatial discretization errors especially in nonsmooth grids. In this paper, a method is developed to analyze the impact of grid discontinuities on the GCL errors and spatial discretization errors. A violation of GCL cause GCL errors which depend on grid smoothness, grid metrics method and finite difference operators. As a result there are more source terms in spatial discretization errors. The analysis shows that the spatial discretization accuracy on nonsufficiently smooth grids is determined by the discontinuity order of grids and can approach one higher order by following GCL. For sufficiently smooth grids, the spatial discretization accuracy is determined by the order of FDSs and FDSs satisfying the GCL can obtain smaller spatial discretization errors. Numerical tests have been done by the second-order and fourth-order FDSs to verify the theoretical results.

AMS subject classifications: 35L04, 65D25, 65D18, 65M06, 65M15

Key words: Geometric conservation law, finite difference scheme, spatial discretization error, nonsmooth grids, high-order accuracy.

1 Introduction

High-order finite difference schemes (FDSs), which can be constructed easily and have high computational efficiency, are widely used in large eddy simulations (LES) and direct

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numerical simulations (DNS) of turbulences and aeroacoustics [1–4]. However, in order to obtain their designed accuracy, FDSs always require sufficiently smooth grids which are very difficult or even impossible to be generated for complex geometries. Complex grids for calculations on practical geometries usually contain nonsmooth features such as slope discontinuities, skewness and stretching, and the impact of these factors on scheme performance is usually significant [5]. Therefore, it is extremely important to study the influence of grid nonsmoothness on spatial accuracy for applications of high-order FDSs to practical engineering problems involving complex geometries.

It has been announced that high-order FDSs can achieve their designed accuracy on sufficiently smooth grids but will degrade their accuracy on non-sufficiently smooth grids. Casper et al. [6] pointed out that a nonsmooth grid will adversely affect the results of finite difference ENO schemes and a sufficiently smooth grid is required to achieve the designed accuracy. Castillo et al. [7] found that the accuracy of fourth-order method degrades gradually as the smoothness of the grid degenerates. Shu [8] also pointed out that the smoothness of meshes must be comparable with the order of accuracy of finite difference WENO schemes in order to obtain a truly high-order result, and the smooth meshes for the fifth-order method mean that at least the fifth derivative of coordinate transformation is continuous. However, the impact of grid smoothness on accuracy of finite difference schemes is usually analyzed numerically but not theoretically.

It was proved that the geometric conservation law (GCL) shall be also satisfied when high-order FDSs are used in the curvilinear coordinates, otherwise some negative effects may appear [9–20], such as violation of free-stream conservation, numerical oscillation. It is true that the GCL is satisfied analytically, but the discrete GCL may be dissatisfied by unsuitable discrete algorithm even on some sufficiently smooth grids, let alone non-sufficiently smooth grids. In recent years, some numerical results showed that satisfying GCL can reduce the requirement on grid smoothness for high-order FDSs, which make it possible to use them to solve problems with complex geometries. In addition, increasing numerical evidences show that satisfying the discrete GCL can improve the time-accuracy of numerical computations and can improve stabilities of these schemes as well (see, e.g., [21–24]). Nonomura et al. [25] showed that the free-stream and vortex preservation properties of WCNS [26–28] is superior to those of the WENO scheme [29], and Deng et al. [30] found the essential reason is that conservative metric method (CMM) can be introduced to ensure WCNS satisfying the GCL. Recently, Nonomura et al. [31] also introduced a technique to make WENO satisfy the GCL and found that with this technique the resolution of vortex is much improved on wavy and random grids. In order to ensure the GCL for high-order finite difference schemes, CMM has been proposed in [30] and a special case of the CMM can be found in Visbal and Gaitonde's numerical technique [5]. Thereafter, symmetrical conservative metric method (SCMM) is derived by analyzing the geometric meaning of metrics and Jacobian [32]. However, the impact of the GCL on spatial accuracy of FDSs has not been analyzed theoretically. For instance, as shown in Refs. [5, 25, 30] correct solutions can be obtained on random grids by FDSs provided that the GCL is satisfied strictly, otherwise, wrong solutions may be yielded if