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A Multilevel Numerical Approach with Application in Time-Domain Electromagnetics

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Abstract. An algebraic multilevel method is proposed for efficiently simulating linear wave propagation using higher-order numerical schemes. This method is used in conjunction with the Finite Volume Time Domain (FVTD) technique for the numerical solution of the time-domain Maxwell's equations in electromagnetic scattering problems. In the multilevel method the solution is cycled through spatial operators of varying orders of accuracy, while maintaining highest-order accuracy at coarser approximation levels through the use of the relative truncation error as a forcing function. Higher-order spatial accuracy can be enforced using the multilevel method at a fraction of the computational cost incurred in a conventional higher-order implementation. The multilevel method is targeted towards electromagnetic scattering problems at large electrical sizes which usually require long simulation times due to the use of very fine meshes dictated by point-per-wavelength requirements to accurately model wave propagation over long distances.

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Key words: Finite volume, time-domain, Maxwell's equations, scattering, multilevel, higher-order.

1 Introduction

Higher-order accurate spatial approximations of partial differential equations (PDEs) are considered necessary in numerical simulations seeking to resolve complex spatial physical phenomena. In principle complex spatial phenomena can be numerically captured with lower-order accurate approximations, but in practice it is often prohibitively expensive to do so because of the large number of grid points required to obtain requisite resolution while operating with lower-order techniques. For numerical simulation of

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scattering problems in time-domain electromagnetics, a basic mesh resolution expressed in terms of points-per-wavelength (PPW) is usually required to adequately resolve the physical process. This mesh resolution stems mainly from the need to accurately model wave propagation both in terms of phase and amplitude over long distances in electromagnetic scattering problems especially at large electrical sizes [1, 2]. The overall mesh size is then dictated by the the electrical size of the problem based on the ratio of the characteristic length of the scatterer and the wavelength of the incident radiation. Numerical techniques like finite difference time domain (FDTD) and finite volume time domain (FVTD) methods are based on directly solving the time domain Maxwell's equations, and face serious difficulties in simulating electromagnetic scattering from electrically large scatterers due to the requirement of long computational times directly related to the fine discretization dictated by PPW requirements [1,3]. Higher-order accurate spatial approximations are usually employed to model the time domain Maxwell's equations with far fewer grid points due to a more relaxed PPW requirement compared to a lower first-order method. This often leads to a reduction in overall computational cost due to a significant decrease in mesh size despite being much costlier on a per-grid-point basis [1, 4]. There have not been many instances in literature of successful advances at an algorithm level to accelerate FDTD or FVTD based numerical simulations to offset this disadvantage of long simulation times at large electrical sizes. The time domain Maxwell's equations solved by FDTD and FVTD methods are a set of linear hyperbolic PDEs, which in the case of the FVTD technique is posed as a set of hyperbolic conservation laws. The multigrid technique [5] in which the solution is cycled through a hierarchy of grids from fine to coarse has been very successful in accelerating convergence of boundary value problems. The multigrid technique initially introduced to solve the system of linear algebraic equations resulting from discretizing linear elliptic PDEs, are based on the efficient smoothing of high frequency error components relative to the discretization in hand. The multigrid method, in the full approximation scheme (FAS) [6] form also applicable to nonlinear problems, uses the relative local truncation error (τ) between finer and coarser grids as a forcing function for coarse grid calculations, in order to maintain fine-grid accuracy for the coarse grid computed solution. The relative truncation error, but between higher and lower order accurate spatial approximations, is also used in defect correction techniques to obtain less expensively higher (usually second) order accuracy by appropriately forcing the lower-order calculation. The concept of τ extrapolation [6] can also be applied to suitably modify the relative truncation error enforced on coarse grid calculations in the multigrid process to get an even higher order accurate approximation on the coarse grid than exists on the finer grid. The original geometric multigrid method based on efficient smoothing of high frequency error components on successive coarser discretizations has been extended, especially in the case of finite element based higher-order discontinuous Galerkin (DG) methods, to the polynomial order or p-multigrid framework based on successive coarser approximations on a fixed discretization. In the p-multigrid method, lower-order polynomial approximations serve as the coarser levels where again high frequency error components are efficiently sought to be smoothed out [7].