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Direct Simulation Methods for Scalar-Wave Envelopes in Two-Dimensional Layered Random Media Based on the Small-Angle Scattering Approximation

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Abstract. This study presents stochastic methods to simulate wave envelopes in layered random media. High-frequency Seismograms of small earthquakes are so complex due to lithospheric inhomogeneity that seismologists often analyze wave envelopes rather than wave traces to quantify the subsurface inhomogeneity. Since the statistical properties of the inhomogeneity vary regionally, it is important to develop and examine direct envelope simulation methods for non-uniform random media. As a simple example, this study supposes plane wave propagation through two-layer random media in 2-D composed of weak and strong inhomogeneity zones. The characteristic spatial-scale of the inhomogeneity is supposed to be larger than the wavelength, where small-angle scattering around the forward direction dominates large-angle scattering. Two envelope simulation methods based on the small-angle scattering approximation are examined. One method is to solve a differential equation for the two-frequency mutual coherence function with the Markov approximation. The other is to solve the stochastic ray bending process by using the Monte Carlo method based on the Markov approximation for the mutual coherence function. The resultant wave envelopes of the two methods showed excellent coincidence both for uniform and for two-layer random media. Furthermore, we confirmed the validity of the two methods comparing with the envelopes made from the finite difference simulations of waves. The two direct envelope simulation methods presented in this study can be a mathematical base for the study of high-frequency wave propagation through randomly inhomogeneous lithosphere in seismology.

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1 Introduction

High-frequency seismic-wave propagation through the earth shows highly complicated behavior because of medium heterogeneity. We see numerous phases in addition to direct P- and S-waves, and S-wave trains are formed around direct S-waves in seismograms of small earthquakes (Fig. 1). Most of the phases are interpreted as waves scattered by random inhomogeneities distributed in lithosphere. Accepting the complexity of the seismograms, seismologists often investigate wave envelopes instead of wave traces to quantify medium inhomogeneity. Observed envelopes show systematic behavior related to the travel distance and the wave frequency [15, 25]. The behavior is strongly governed by the statistical properties of the medium inhomogeneity.

In seismology, there are some approaches for simulating directly wave envelopes without calculating wave traces based on the statistical information such as the power spectrum density function (PSDF) or the auto correlation function (ACF) of medium fractional fluctuation. We refer to such approaches as direct envelope simulation methods. The radiative transfer theory is one of the methods to simulate the envelopes of coda waves [e.g., 1, 3]. The heuristic approach to the radiative transfer theory and its application to seismology are summarized in [28]. Theoretical derivation of the radiative transfer equation based on the Bethe-Salpeter equation for waves through random media is well summarized in [12]. The validity of the radiative transfer theory with the Born approximation is examined carefully by a comparison with numerical finite-difference simulations [18, 38].

Another typical method for the direct envelope simulation is based on a statistical method called the Markov approximation which supposes small-angle scattering around the forward direction. This method was originally developed for optical waves in radio physics [9, 20, 32, 36], and may be considered as a stochastic extension of the split step method for the parabolic wave equation [8, 34]. The validity of the Markov approximation can be examined by a comparison between the envelope of the Markov approximation and the averaged envelopes which are made from the numerically calculated wave traces [2]. Employing the method of the Markov approximation, H. Sato simulated envelopes of S-wave trains, or S-wave envelopes, and showed the duration of the envelopes increases with increasing the travel distance [25]. It is consistent with the observation that S-wave trains of small earthquakes usually show the increase in the duration as shown in Fig. 1. Following this study, many studies have applied the method for the interpretation of high-frequency seismograms [15, 16, 24, 30, 35]. In parallel with the application to observed seismograms, theoretical simulation methods have also been developed to adopt more realistic conditions for the lithospheric inhomogeneity. The original method