Commun. Comput. Phys.
doi: 10.4208/cicp.091014.030515a

Vol. **21**, No. 2, pp. 443-465
February 2017

An Alternative Lattice Boltzmann Model for Incompressible Flows and its Stabilization

Liangqi Zhang[1,2], Zhong Zeng[1,3,*], Haiqiong Xie[1], Zhouhua Qiu[1], Liping Yao[4], Yongxiang Zhang[1] and Yiyu Lu[2]

[1] *Department of Engineering Mechanics, College of Aerospace Engineering, Chongqing University, Chongqing 400044, P.R. China.*
[2] *State Key Laboratory of Coal Mine Disaster Dynamics and Control, Chongqing University, Chongqing 400044, P.R. China.*
[3] *State Key laboratory of Crystal Material, Shandong University, Jinan 250100, P.R. China.*
[4] *College of Engineering and Technology, Southwest University, Chongqing 400716, P.R. China.*

Communicated by Kun Xu

Received 9 October 2014; Accepted (in revised version) 3 May 2015

Abstract. In this paper, an alternative lattice Boltzmann (LB) model for incompressible flows is proposed. By modifying directly the moments of the equilibrium distribution function (EDF), the continuous expression of the EDF in tensor Hermite polynomials is derived using the moment expansion and then discretized with the discrete velocity vectors of the D2Q9 lattice. The present model as well as its counterpart, the incompressible LB model proposed by Guo, reproduces the incompressible Navier-Stokes (N-S) equations for both steady and unsteady flows. Besides, an alternative pressure formula, which represents the pressure as the diagonal part of the stress tensor, is adopted in the present model. Furthermore, in order to enhance the stability of the present LB model, an additional relaxation time pertaining to the non-hydrodynamic mode is added to the BGK collision operator. The present LB model is validated by two benchmark tests: the cavity flow with different Reynolds number (Re) and the flow past an impulsively started cylinder at Re = 40 and 550.

PACS: 47.10.ab, 47.11.-j, 02.60.Cb

Key words: Lattice Boltzmann method, incompressible flows, cavity flow, deviatoric stress, compressibility effect.

*Corresponding author. *Email addresses:* `liangqizhang@cqu.edu.cn`, `liangqizhang@ntu.edu.sg` (L. Zhang), `zzeng@cqu.edu.cn` (Z. Zeng), `xiehaiqiong@cqu.edu.cn` (H. Xie), `qiuzhouhua@cqu.edu.cn` (Z. Qiu), `lpyao@swu.edu.cn` (L. Yao), `zhangyx@cqu.edu.cn` (Y. Zhang), `luyiyu@cqu.edu.cn` (Y. Lu)

1 Introduction

The LB method, a promising alternative to conventional numerical methods for fluid dynamics, has win great popularity owing to its successful application in complex fluids such as suspensions and interfacial dynamics [1–6]. The LB method is essentially a second-order explicit finite difference approximation of the relevant macroscopic equations [7, 8]. The superior features of LB method over its second order finite difference counterparts based on direct discretization of the Navier-Stokes (N-S) equations are its low numerical dissipation and better isotropy [1, 9, 10].

The LB method consists of two core components: the lattice Boltzmann equation (LBE), governing the evolution of the distribution function, and the relationships, represented by the moment equations of the distribution function, between the distribution function and the macroscopic variables. By applying the Chapman-Enskog expansion [11–13], the macroscopic N-S equations can be derived from the LBE, and the specific forms of the relevant macroscopic equations largely depend on the moments of the equilibrium distribution function (EDF). With the prescribed EDF of the standard LB model, the isothermal, compressible N-S equations are obtained, and the solutions of incompressible equations are approximated by this model with second order error in Mach number (*Ma*) [9]. In order to reduce the compressibility error in the standard LB model, an incompressible LB model, notated as He-Luo model, is proposed by He and Luo [14,15], in which the fluid density ρ is replaced by a constant mean density ρ_0 everywhere ρ is multiplied by the fluid velocity $\mathbf{u}$. The macroscopic equations derived from this model approximate the incompressible N-S equation with $\mathcal{O}(Ma^3)$ error for steady flow. However, for unsteady flow, the compressibility error associated with the continuity equation remains $\mathcal{O}(Ma^2)$ [9, 14]. Based on the assumptions that the fluid density is a uniform constant and the fluid pressure is independent of density, another incompressible LB model was designed by Guo in 2000 [16]. And the resulting macroscopic equations pertaining to Guo's model are incompressible for both steady and unsteady flows. Although the error of the corresponding momentum equation is of the order $\mathcal{O}(Ma^2)$, the compressibility error is eliminated by setting the fluid density to be a constant.

In this work, inspired from Guo's model, an alternative incompressible LB model is proposed by using Grad's moment expansion [17, 18]. The present model inherits the advantageous features of Guo's model, i.e. the compressibility error associated with the continuity equation vanishes. Besides, an alternative pressure formula, which accords with the definition of the mechanical pressure [12], is adopted in the present model and provides more accurate evaluation of the deviatoric stress than Guo's model. Moreover, the equivalent moment systems in the diffusive scale, derived by M. Junk [7, 8], demonstrate that the truncated error term, $\Delta\rho$, linked to the Laplacian of pressure for the standard LB model and the He-Luo model, has explicit effect on the mass conservation and the accuracy of the modeled stress tensor [19]. Theoretically, with the basic assumption that the fluid density is a uniform constant, the present model and the Guo's model are expected to reduce such error.