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Analysis of the Effect of Spatial Uncertainties on the Dynamic Behavior of Electrostatic Microactuators

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Abstract. This paper examines the effect of spatial roughness on the dynamical behaviour of electrostatic microactuators. We develop a comprehensive physical model that comprises a nonlinear electrostatic actuation force as well as a squeeze-film damping term to accurately simulate the dynamical behavior of a cantilever beam actuator. Spatial roughness is modeled as a nonstationary stochastic process whose parameters can be estimated from profilometric measurements. We propagate the stochastic model through the physical system and examine the resulting uncertainty in the dynamical behavior that manifests as a variation in the quality factor of the device. We identify two distinct, yet coupled, modes of uncertainty propagation in the system, that result from the roughness causing variation in the electrostatic actuation force and the damping pressure, respectively. By artificially turning off each of these modes of propagation in sequence, we demonstrate that the variation in the damping pressure has a greater effect on the damping ratio than that arising from the electrostatic force. Comparison with similar simulations performed using a simplified mass-spring-damper model show that the coupling between these two mechanisms can be captured only when the physical model includes the primary nonlinear interactions along with a proper treatment of spatial variations. We also highlight the difference between nonstationary and stationary covariance formulations by showing that the stationary model is unable to properly capture the full range of variation as compared to its nonstationary counterpart.

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Key words: Uncertainty quantification, microelectromechanical systems (MEMS), electrostatic actuation, squeeze-film damping, roughness, random process, damping ratio.

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1 Introduction

The accurate dynamic analysis of micromechanical actuators has been the subject of a lot of research and have led to the development of precise electromechanical resonators [1–4]. Due to the nature of the manufacturing process involved in their fabrication, microsystem designers have to account for several uncertainties in the form of variation in the properties of these devices [5, 6]. The study of these uncertainties is important for understanding their effect on the performance of a device as well as in modifying its design to make it robust towards these variations [7–9]. In this paper, we are concerned with the outcome of spatial variations in dimensional parameters on the dynamical behavior of microelectromechanical systems (MEMS). We focus on systems like oscillators and resonators that are driven using electrostatic actuation, since the electric field that is used for actuation in these systems, is particularly sensitive to spatial perturbations in the geometry of the device. Using stochastic models to establish a formal description of spatial uncertainties, we try to predict the resulting variation in the quality factor associated with these resonators, which is a relevant performance metric. The goal is to use a proper physical model in order to identify the dominant mode of uncertainty propagation in such devices and thereby understand the role that spatial variations play in them.

A simple electrostatic microactuator can be modeled as a pair of conducting electrodes, one of which is held fixed while the other is allowed to move. A potential difference applied between the two electrodes creates an electric field that results in an attractive electrostatic force. The movable electrode is either attached to a mechanical spring or is compliant enough to deform under the action of the applied force, while being anchored at one or more locations, and is responsible for converting the electrostatic force into a displacement [10–12]. In addition to the mechanical restoring force offered by the compliant structure, the motion of the movable electrode is also retarded by various damping phenomena. At meso- and micro-scales, the principle cause of damping is the surrounding medium, which is typically air. This can be suitably modeled using squeeze film damping theory, where the film of air that gets trapped between the electrodes, when the device oscillates at relatively high frequencies, causes a back-pressure that retards the motion of the system [13]. The magnitude of the electrostatic force is intimately linked with the nature of the electric field generated between the electrodes. As such, any spatial variation of the surfaces of these electrodes will lead to changes in the electric field and ultimately, the displacement as well. Spatial roughness also affects the damping force, since it modifies the flow of air in the squeeze-film region. This can affect the dynamic behavior of electrostatic oscillators operating at high frequencies and can be quantified by observing the variation in the quality factor, which is a measure of ratio of energy stored in an oscillator to the energy that is dissipated [14].

Surface variations in MEMS are usually modeled either using stochastic processes, where the roughness is specified in terms of a local covariance structure [6, 15], or using self-affine roughness models, where the roughness spectrum is assumed to have a power law scaling [16, 17]. The effect of spatial roughness on the dynamical behavior of