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Numerical Simulations of Particle Sedimentation Using the Immersed Boundary Method

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Abstract. We study the settling of solid particles in a viscous incompressible fluid contained within a two-dimensional channel, where the mass density of the particles is greater than that of the fluid. The fluid-structure interaction problem is simulated numerically using the immersed boundary method, where the added mass is incorporated using a Boussinesq approximation. Simulations are performed with a single circular particle, and also with two particles in various initial configurations. The terminal particle settling velocity and drag coefficient correspond closely with other theoretical, experimental and numerical results, and the particle trajectories reproduce the expected behavior qualitatively. In particular, simulations of a pair of interacting particles similar drafting-kissing-tumbling dynamics to that observed in other experimental and numerical studies.

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Key words: Immersed boundary method, particle suspension, sedimentation, settling velocity, fluid-structure interaction.

1 Introduction

Particulate flows involve the interaction of a dynamically evolving fluid with solid suspended particles, and arise in a wide range of applications in natural and industrial processes [10]. We are particularly interested in the gravitational settling or sedimentation problem, in which the suspended particles settle under their own weight. Sedimentation is observed in many applications including flow of pollutants in rivers and the atmosphere, tea leaves settling in a teacup, industrial crystal precipitation, mineral ore processing, and hail formation in thunderclouds to name just a few.

There is an extensive literature on experimental, theoretical and computational studies of particulate flows. We will not attempt to provide a comprehensive review here, but

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will rather highlight a few of the more important results. Experimental studies of sedimentation have had a long history including the early work of Richardson and Zaki [40] and extending to more recent years [15,19,27,28]. Many analytical and approximate solutions have been developed that capture the behavior of settling suspensions, especially in the dilute limit where there are only a small number of particles. Back in 1851, Stokes [43] derived an analytical solution for a single particle settling within an unbounded fluid, and many other authors have since extended these results to other more practical sedimentation problems [7, 14, 47]. More recently, many numerical approaches have been developed to simulate settling particles, including the finite element [18, 21, 35], lattice-Boltzmann [16, 31], and boundary element methods [25, 38]. The common feature in all of these approaches is that the fluid flow is governed by the Navier-Stokes equations whereas the particles are governed by Newton's equations of motion. The hydrodynamic forces between the particle and fluid are obtained from the solution of this coupled system, which typically requires either complex interfacial matching conditions at the fluid-particle interface or else some form of dynamic boundary-fitted meshing. In any case, these methods tend to be complex and extremely CPU-intensive, especially for three-dimensional problems.

One numerical approach that has proven to be especially effective for solving complex fluid-structure interaction problems involving dynamic moving structures is the *immersed boundary* (or IB) method. This approach has been used extensively to simulate deformable structures arising from problems in biofluid mechanics [37]. For example, Wang and Layton [48] have used the IB method to simulate sedimentation of multiple rigid 1D fibers suspended in a viscous incompressible fluid, while several other authors have applied the IB approach to solve related sedimentation problems [8, 16, 26, 46, 49].

The IB method is a mixed Eulerian-Lagrangian approach in which the fluid equations are solved on an equally-spaced rectangular mesh, while the moving boundaries are approximated at a set of points that moves relative to the underlying fluid grid. In the original IB method, the effect of these immersed boundaries is represented as a singular force that is computed from the current IB configuration and spread onto fluid grid points by means of a regularized delta function. The added mass due to a sedimenting particle can be distributed onto the fluid in a similar manner. With the exception of the papers by Wang and Layton [48] and Hopkins and Fauci [26], the other authors mentioned above have employed a modification of this IB approach known as the "direct forcing IB method." In this approach the force instead takes the role of a Lagrange multiplier that is chosen to constrain the fluid and structure to move with the same velocity (see [34] for more details). The direct forcing method has the advantage that it resolves the immersed structure sharply and avoids the sometimes severe time step restriction arising from very stiff springs; on the other hand, it is also considerably more complicated to implement, especially when the immersed boundary is moving.

Our aim in this paper is apply the original IB method rather than the direct forcing approach to solving sedimentation problems. We restrict ourselves to a two-dimensional geometry in which one or two particles with circular cross-section settle under the influ-