Commun. Comput. Phys.
doi: 10.4208/cicp.230113.140814a

Vol. **17**, No. 2, pp. 458-486
February 2015

Mixed Spectral Element Method for 2D Maxwell's Eigenvalue Problem

Na Liu[1], Luis Tobón[2,3], Yifa Tang[4] and Qing Huo Liu[2,*]

[1] *Institute of Electromagnetics and Acoustics, Xiamen University, Xiamen, 361005, P.R. China.*
[2] *Department of Electrical and Computer Engineering, Duke University, Durham, NC, 27708, USA.*
[3] *Departamento de Ciencias e Ingeniería de la Computación, Pontificia Universidad Javeriana, Cali, Colombia.*
[4] *LSEC, ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, P.R. China.*

Received 23 January 2013; Accepted (in revised version) 14 August 2014

Communicated by Wei Cai

Abstract. It is well known that conventional edge elements in solving vector Maxwell's eigenvalue equations by the finite element method will lead to the presence of spurious zero eigenvalues. This problem has been addressed for the first order edge element by Kikuchi by the mixed element method. Inspired by this approach, this paper describes a higher order mixed spectral element method (mixed SEM) for the computation of two-dimensional vector eigenvalue problem of Maxwell's equations. It utilizes Gauss-Lobatto-Legendre (GLL) polynomials as the basis functions in the finite-element framework with a weak divergence condition. It is shown that this method can suppress all spurious zero and nonzero modes and has spectral accuracy. A rigorous analysis of the convergence of the mixed SEM is presented, based on the higher order edge element interpolation error estimates, which fully confirms the robustness of our method. Numerical results are given for homogeneous, inhomogeneous, L-shape, coaxial and dual-inner-conductor cavities to verify the merits of the proposed method.

AMS subject classifications: 78M10, 78M22

Key words: Maxwell eigenvalues, spectral element method, spurious eigenvalues.

*Corresponding author. *Email addresses:* liuna@xmu.edu.cn (N. Liu), let12@duke.edu (L. Tobón), tyf@lsec.cc.ac.cn (Y. Tang), qhliu@duke.edu (Q. H. Liu)

1 Introduction

The computation of eigenvalues in Maxwell's equations is of fundamental importance in computational electromagnetics. Since nodal based finite element method was introduced to electrical engineering by Silvester in 1969 [1], it has been shown that the nodal based finite element method has several advantages in solving homogeneous waveguide problems, but in solving inhomogeneous waveguide problems it yields spurious modes with nonzero eigenvalues. Much effort has been made to reduce or eliminate this unwanted numerical behavior due to the presence of spurious modes [2]. Rahman and Winkler observed that spurious modes do not satisfy the zero divergence condition on the electric or magnetic field and suggested a penalty function to enforce this condition [3–5]. Unfortunately, this method cannot eliminate the spurious modes completely and leaves the user the task of selecting a suitable penalty function parameter.

The other approach to eliminating spurious modes in the finite element waveguide problem is to find proper finite element approximation functions [2,6–9]. It is now well-known that employing $\mathbf{H}(\mathrm{curl};\Omega)$-conforming basis functions (also known as edge elements) for the electric field [7,8] can ensure the continuity of tangential field components across an interface between different media, while leaving the normal field components free to jump across such interfaces. With the edge element method there are no spurious modes with nonzero eigenvalues, but the number of spurious modes with zero eigenvalue is equal to the number of nodal points inside the computational domain due to the violation of Gauss's law [10]. Therefore, in order to remove these zero eigenvalues, in addition to using the proper finite element space, the divergence free property (Gauss's law) of the eigenfunction must be enforced. One such successful approach for computing eigenvalues in waveguide problems is to employ $\mathbf{H}(\mathbf{curl};\Omega)$-conforming basis functions to approximate the electric field while imposing the divergence-free condition through the use of a Lagrange multiplier, as suggested by Kikuchi [14].

In the meantime, higher-order methods such as the spectral element method (SEM) [18,19] have also been proposed to solve electromagnetic eigenvalue problems. But these methods also suffer from the presence of spurious zero eigenvalues, even though these methods have a high convergence rate. In this paper, we present the mixed spectral element method (mixed SEM) by applying the divergence free equation in Kikuchi's scheme into the SEM, for which the preliminary results have been shown in [20]. In the SEM, the basis functions are constructed by Gauss-Lobatto-Legendre (GLL) polynomials [18,19]. The accuracy increases exponentially with increasing the order of GLL basis functions, while the number of degrees of freedom increases slowly. The convergence of this higher-order mixed SEM is proved under the stronger regularity assumptions based on the interpolation estimates for GLL edge elements and the abstract spectral theory in mixed form [10,11,13,23]. We apply this mixed spectral element method to the two-dimensional vector Maxwell TEz eigenvalue problem and carry out numerical experiments on a homogeneous cavity, an inhomogeneous cavity, a L-shape singular cavity, a coaxial cavity and a two PEC cavity to validate this method. The results show that the mixed spec-