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An Efficient Spectral Method for Acoustic Scattering from Rough Surfaces

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Abstract. An efficient and accurate spectral method is presented for scattering problems with rough surfaces. A probabilistic framework is adopted by modeling the surface roughness as random process. An improved boundary perturbation technique is employed to transform the original Helmholtz equation in a random domain into a stochastic Helmholtz equation in a fixed domain. The generalized polynomial chaos (gPC) is then used to discretize the random space; and a Fourier-Legendre method to discretize the physical space. These result in a highly efficient and accurate spectral algorithm for acoustic scattering from rough surfaces. Numerical examples are presented to illustrate the accuracy and efficiency of the present algorithm.

AMS subject classfications: 65C20, 65C30, 78A45, 35J05

Key words: Acoustic scattering; spectral methods, stochastic inputs, differential equations, uncertainty quantification.

1 Introduction

Wave scattering from (random) rough surfaces is a very common physical phenomenon and is at the core of a variety of technologies ranging from SAR imaging and remote sensing to underwater acoustics, optical lithography, and meteorology. The design of numerical methods to faithfully simulate such scattering processes presents significant challenges, largely due to the need to resolve wave oscillations and interactions, and in many cases, high accuracy requirements.

A wide variety of techniques have been proposed for wave scattering from irregular obstacles, e.g., small-slope approximation, Kirchhoff approximation, momentum transfer expansion, etc. (see the survey paper [25] and the references therein). Among the most compelling of these are those based on boundary perturbations which can be traced

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to the work of Rayleigh [16] and Rice [17]. These methods are fast, straightforward to implement, and quite accurate within their domain of applicability. It has been shown rigorously that the scattered fields are analytical functions of the grating height (slope) parameter ([2,8]), and this provides justifications for boundary perturbations. However, the traditional approaches of [16,17] become *ill-conditioned* at high orders because they rely on significant cancellations for convergence ([12,13]). Subsequently, an improved boundary perturbation algorithm was proposed in [13] to overcome the issue of poor conditioning. This method, termed "Transformed Field Expansions" (TFE), employs a change of variables which "flattens" the shape of the scatter to a flat configuration and is shown to be accurate and robust at high orders. In [14], the TFE technique is combined with a highly efficient spectral Galerkin method in polar coordinates and applied to bounded obstacles. Extensive numerical experiments were conducted to demonstrate the efficiency and accuracy of the method.

Due to the lack of knowledge and/or measurement of realistic rough surfaces, it is natural to adopt a probabilistic framework, where rough surfaces are modeled as random processes which are typically characterized by surface height probability distributions and their cross correlations. Most of the existing numerical approaches adopt one of the afore-mentioned methods for a single deterministic irregular surface. Statistical averaging is then applied over an ensemble of such deterministic simulations to generate probabilistic information of the scattered fields, e.g., mean, standard deviation, etc. This is the classical Monte Carlo type procedure and can be computationally intensive as the convergence rate of the statistics is relatively slow, e.g., the mean field usually converges as $1/\sqrt{K}$ where K is the number of realizations. A large solution ensemble involving many realizations of deterministic simulations are required to obtain sufficiently accurate solution statistics. Recently, a high-order non-sampling method based on Generalized Polynomial Chaos (gPC) was developed for differential equations with random inputs [29]. It is a generalization of the Wiener-Hermite expansion [26] and is essentially a spectral approximation of functionals in random space. The basis functions include global orthogonal polynomials [4, 5, 29], piecewise polynomials [1], and wavelets [6]. Along with Galerkin projection, the resulting stochastic Galerkin (SG) algorithm can be highly efficient in many applications with random initial/boundary conditions, uncertain parameters, etc. See, for example, [28,30] for numerical examples, [1] for analysis on elliptic equations and [3] on Burgers' equation.

Stochastic modeling in (random) rough domains, despite its practical and theoretical importance, is relatively underdeveloped. Efficient numerical algorithms using gPC expansions were developed in [31] and [24], where a robust numerical mapping and an analytical mapping, respectively, were employed to transform the problems into fixed domains. In this paper, we extend the TFE boundary perturbation method to scattering of *random rough surfaces*. The surfaces are to be modeled as random processes and the resulting governing equations, Helmholtz equation for acoustic scattering, become stochastic partial differential equations. The gPC expansion is employed as a spectral method in random space, and a Galerkin method is used to obtain the governing equations for the