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A 3D Numerical Method for Studying Vortex Formation Behind a Moving Plate

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Abstract. In this paper, we introduce a three-dimensional numerical method for computing the wake behind a flat plate advancing perpendicular to the flow. Our numerical method is inspired by the panel method of J. Katz and A. Plotkin [J. Katz and A. Plotkin, Low-speed Aerodynamics, 2001] and the 2D vortex blob method of Krasny [R. Krasny, *Lectures in Appl. Math.*, 28 (1991), pp. 385–402]. The accuracy of the method will be demonstrated by comparing the 3D computation at the center section of a very high aspect ratio plate with the corresponding two-dimensional computation. Furthermore, we compare the numerical results obtained by our 3D numerical method with the corresponding experimental results obtained recently by Ringuette [M. J. Ringuette, Ph.D. Thesis, 2004] in the towing tank. Our numerical results are shown to be in excellent agreement with the experimental results up to the so-called formation time.

Key words: Aerodynamics; vortex dynamics; potential flows.

1 Introduction

This work is inspired by our desire to develop a 3D computational method to study the fluid dynamics of flying animals like birds or fish. Flight has fascinated the human mind since its early days of development. The gracefulness and the perfection of a bird in the air inspired us to try to emulate some particular aspects of its superb ability. The fluid dynamics in the flight of a bird is extremely complex and interesting. In particular, we are interested in understanding the way that the wake is generated and its effect on the

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wing. A number of computational methods have been developed to study fluid dynamics of the motion of a moving surface. However, it is still a challenge to develop an accurate and efficient three dimensional computational method which can be used to simulate the flight of a bird.

There have been many theoretical and computational studies to understand the fluid dynamics instability for flow past a flexible object. Starting with the work of Sir James Gray who was the head of Cambridge University's Zoology Department from 1937 to 1961, the theory of animal locomotion stimulated G. I. Taylor in making two pioneering investigations. One involved the swimming of snakes and eels, and the other one initiated hydrodynamic studies of flagellar propulsion. However, the main contributions to this domain came later on from Sir James Lighthill and Professor Theodore Wu. Lighthill laid a theoretical foundation for the swimming of slender fish, while Wu made an extension of classical oscillating airfoil aerodynamics to a linear theory of flexible lifting-surface locomotion, to examine the performance of bending wings of birds in flapping flight and the lunate tails of fast-swimming percomorph and acombroid fishes and rays in swimming.

Re-emphasizing the importance of Wu's (scaling of aquatic locomotion-[21]), Lighthill's (scaling of aerial locomotion) and Weis-Fogh's (hovering flight) work in fish and bird locomotion, we should also review the advances made in the field of wing theory and free-surface numerical methods. One of the pioneers in the study of a wing in flight was T. Theodorsen with his theory of instability and the mechanism of flutter [18]. In 1931, Kaden introduced similarity variables for describing the roll-up of a semi-infinite plane vortex-sheet ([17], p. 147). During the 1950s, several authors presented approximate asymptotic solutions in which the spiral vortex generated behind a sharp edge was replaced by a single point vortex, an example of which was given by [16]. Moore in 1976 [12] studied and verified the stability of a class of vortex sheets roll-up, but could not verify the stability of the problem as noted by Pullin in 1978 [14]. In the same work of Pullin, he obtained for the first time regular and well-defined start-up vortex spirals from an accurate numerical solution of the Birkhoff-Rott equation (inviscid flow) written in similarity variables. These results were later used by Krasny to validate his method.

In 1991, Krasny [10] generalized Chorin's vortex blob method [3] and applied it to study the evolution of the wake forming at the edges of a flat plate advancing perpendicular to the flow. Moreover, he presented strong numerical evidence suggesting that the vortex blob method converges past the vortex sheet singularity formation time, as the smoothing parameter tends to zero. Later on, in 1994, Nitsche and Krasny [13] generalized the vortex blob method to study the vortex ring formation at the edge of a circular tube. Comparison between the numerical simulation and the corresponding experiment indicated that the model captured the basic features of the ring formation process. Then, in 2001, a very interesting model was presented by Chamara and Coller [2] for a pair of airfoils with two degrees of freedom: pitching and heaving. Their study sheds interesting light in finding the optimal configuration of the two-airfoil system for which the two oscillatory instabilities due to the flutter occur simultaneously.

In the study of animal locomotion, Dickinson *et al.* designed interesting experiments to