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An Adjoint-Based h-Adaptive Reconstructed Discontinuous Galerkin Method for the Steady-State Compressible Euler Equations

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Abstract. In this paper, an adjoint-based h-adaptive high-order reconstructed DG (rDG) method is introduced for solving the two dimensional steady-state compressible Euler equations. Based on the hybrid reconstruction strategy developed in [9,28], adjoint-based a posteriori error estimation is further derived and developed for h-adaption. The formulation of error indicator is carefully investigated in order to deliver better approximation with respect to the error in the computed output functional. In order to evaluate the performance of the proposed method, an adjoint-based h-adaptive rDG(p_1p_2) method is implemented, in which a hybrid p_1p_2 reconstruction and a hybrid p_2p_3 reconstruction are adopted in the primal solver and the adjoint solver to obtain the primal solution and the adjoint solution, respectively. A number of typical test cases are selected to assess the performance of the adjoint-based h-adaptive hybrid rDG method. The hybrid reconstruction strategy combined with h-adaptive techniques based on adjoint-based error estimate presented in this work demonstrates its capacity in reducing the error with respect to the computed output functional and improving the level of accuracy for numerical simulations of the compressible inviscid flows.

AMS subject classifications: 65M60, 65M99, 35L65

Key words: Reconstructed discontinuous Galerkin method, adjoint-based error estimate, h-adaptivity, compressible Euler equations

1 Introduction

The last two decades have witnessed considerable progress in developing and applying discontinuous Galerkin (DG) methods [1–4, 8, 10, 16, 21, 38, 40] in the area of computa-

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tional fluid dynamics. The DG methods combine two advantageous features commonly associated to finite element (FE) and finite volume (FV) methods. Nevertheless, the DG methods still have their own weaknesses. Compare to the classical FE or FV methods, the DG methods usually require more degrees of freedom, which leads to higher computational costs and storage requirements.

One feasible approach to reduce the high computational costs leads to the development of reconstructed DG (rDG) methods, termed as $p_n p_m$ schemes [11, 12], where p_n indicates a piecewise polynomial of degree n to approximate a DG solution, and p_m represents a reconstructed polynomial solution of degree m ($m > n$) to compute the fluxes. The $p_n p_m$ strategy is designed to enhance the accuracy of the underlying DG(p_n) method by increasing the order of the solution polynomial through reconstruction while at the same time to reduce computational costs and storage requirements by decreasing the number of degrees of freedom compared to the traditional DG(p_m) formulation. Obviously, an accurate and efficient reconstruction operator is crucial in the construction of the $p_n p_m$ schemes.

Several ways exist for designing the reconstruction operator based on the DG framework. Normally, this is achieved using a so-called in-cell recovery approach, i.e., the $p_n p_m$ schemes, where the recovered piecewise polynomial solution is uniquely determined by making it indistinguishable from the underlying DG solutions in the contributing cells based on a weak sense. It should be noted that, similar to the in-cell recovery strategy, an inter-cell recovery DG method have been introduced and developed by van Leer et al. [13, 14], which is designed for solving diffusion problems. Luo et al. [5, 6, 29, 41] developed a reconstructed DG (rDG) method using Taylor basis [3] for simulating compressible flows on arbitrary grids, where a higher-order polynomial solution is reconstructed through a strong interpolation, requiring point values and derivatives to be interpolated on the face-neighboring cells. This reconstruction scheme only involves the von Neumann neighborhood, thus is compact, simple, robust, and flexible. In order to further enhance the accuracy of the reconstructed DG method for viscous flows, a hybrid reconstruction DG method, which can be regarded as a combination of the recovery strategy and reconstruction strategy, has been developed by Cheng et al. [9, 28]. Zhang et al. [15–17] introduced a class of hybrid DG/FV methods using a Green-Gauss reconstruction which is commonly used in the FV methods. This provides a fast, simple, and robust way to obtain higher-order polynomial solutions. However, this reconstruction sometimes becomes less accurate, since it does not guarantee the property of k-exactness [18, 23].

From another perspective, the solutions of nonlinear hyperbolic conservation laws, such as the compressible Euler equations, often exhibit a wide range of localized structures which can be difficult for numerical methods to capture these features accurately based on a fixed mesh without a large number of mesh cells. However, it usually becomes both computationally intensive and time consuming if a globally refined mesh is used. Thus, the use of appropriate adaptive strategies, which has shown a strong ability in reducing computational cost for a variety of real applications [19], becomes highly desirable