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A Second-Order Two-Scale Algorithm for Thermo-Mechanical Coupling Problems in Quasi-Periodic Porous Materials

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Abstract. This work develops a second-order two-scale (SOTS) model based on homogenization method to predict thermo-mechanical coupling performance of porous materials with quasi-periodic structures. For the kinds of porous materials, the corresponding material coefficients are dependent on the macroscopic variable and the radiation effect at microscale is considered in this paper. The quasi-periodic properties of the thermo-mechanical coupling models which consider mutual interaction between temperature and displacement fields are proposed at first. Then, the two-scale formulas for the thermo-mechanical coupling problems with radiation boundary conditions are derived successively, and the finite element algorithms based on the SOTS model are brought forward in detail. Finally, by some typical examples, the effectiveness and validity of the proposed algorithms are confirmed. The computational results demonstrate that the SOTS method is efficient and valid to predict the thermo-mechanical coupling properties, and can acquire the microscale information of the porous materials accurately.

AMS subject classifications: 35B27, 34E13, 74Q05, 83C30

Key words: SOTS model, radiation effect, thermo-mechanical performance, quasi-periodic structures.

1 Introduction

In recent decades, porous materials are extensively applied in civil engineering, aerospace and military industry etc. Such applications are mainly due to the good thermal stabil-

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ity, low relative density and high heat resistance. In particular, porous materials have been manufactured to be used in thermal protection system (TPS), which have caused widespread research interests in aerospace industry. In general, as the space vehicle entering or flying out of the atmosphere, the surface of aerocraft will endure strong aerodynamic heating and the surface temperature will be high enough in a moment. In this case, the thermal-mechanical problems of porous materials will be significantly sophisticated owing to the main influence of heat radiation. Further, since the composites usually have periodic structure and material parameters oscillate fast in microscale cell, a highly efficient numerical method should be established to predict the thermal-mechanical coupling performance of porous materials with periodic distributions.

Radiation is a form of heat transfer and plays a significant role in heat transmission, especially under a high temperature environment. Some representative works have been given to simulate the heat radiation performance of porous materials [1–6]. Recently, Yang et al. [7, 8] discussed the heat radiation problems in periodic porous materials, and developed higher-order multiscale expansions for the coupled problems. Ma and Cui [9] and Allaire and Ganaoui [10] investigated the coupled conduction and radiation problems by homogenization method and two-scale convergence method for high porosity materials, respectively. In addition, thermo-mechanical behavior of composites with rapidly varying coefficients is described by the coupling between a hyperbolic equation and a parabolic equation, and some scientists and engineers have contributed to extensive research results for the problems [11–25]. Temizer and Wriggers [11] gave some detailed theoretical results for the linear thermoelasticity problems by homogenization method. Feng and Cui [12] constructed a SOTS model for stationary thermo-mechanical problem of composites with rapid oscillation coefficients. On the basis of work in Cui et al. [12], Wan [13] and Yang and Cui [14] applied the SOTS model and multiscale theory to analyze the dynamic thermo-mechanical coupling performance of composites with small scale ε, and developed a higher-order finite element algorithm. Terada et al. [15] reported a scale effect of heat convection in unit cells, and obtained first-order multiscale formulas for the thermo-mechanical problems in periodic composite materials. Zhang et al. [16] and Yu and Tang [17] considered the thermo-mechanical problem arising from periodic composites using multiscale methods and ignored the coupling term to obtain a simplified problem. Francfort [18] proposed a homogenized method for the dynamic thermo-mechanical coupling problems of periodic composites, and obtained some meaningful theoretical results. Furthermore, Brahim-Otsmane et al. [19] developed the homogenized method to analyze the thermo-mechanical properties of the random microstructures, and can accurately estimate the effective macroscopic modulus. Later, Vel et al. [20] and Goupee and Vel [21] presented a novel multiscale analysis based on traditional homogenization method for the thermo-mechanical coupling problems of random inhomogeneous composite materials. Aboudi et al. [26] considered the thermo-mechanical coupling problems based on a novel micromechanics model, which can accurately predict the elastic modulus, and effectively capture stress and strain fields in the periodic composites. Yu and Fish [27] discussed a systematic homogenization method to inves-