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An Adaptive Finite Element PML Method for the Acoustic Scattering Problems in Layered Media

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Abstract. The paper concerns the numerical solution for the acoustic scattering problems in a two-layer medium. The perfectly matched layer (PML) technique is adopted to truncate the unbounded physical domain into a bounded computational domain. An a posteriori error estimate based adaptive finite element method is developed to solve the scattering problem. Numerical experiments are included to demonstrate the efficiency of the proposed method.

AMS subject classifications: 65N30, 78M10, 35Q99

Key words: Acoustic scattering problems, layered media, perfectly matched layer, adaptive finite element method.

1 Introduction

Numerical solutions of scattering problems have drawn considerable attention in both the engineering and mathematical communities. The first key point of numerical solutions is the treatment of radiation conditions at infinity. It involves the truncation of an unbounded domain to a bounded domain and imposes highly accurate boundary conditions at the artificial boundary (cf. e.g. [24–26, 40]). Scattering problems involving infinite boundaries, such as the scattering in layered media and half-spaces (cf. e.g. [9, 17, 20, 22, 41]), are studied recently. With the appearance of infinite boundaries, the scattering waves usually comprise reflective waves and evanescent waves. Hence the numerical treatment of radiation conditions becomes very challenging and appeals for new theories and methods.

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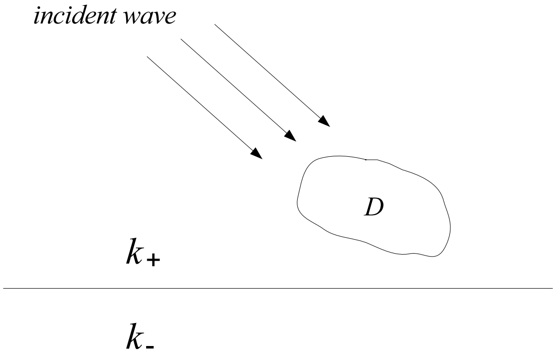

Figure 1: A schematic of the geometry for the scattering problem in a two-layer media.

In this paper, we study the two-dimensional acoustic scattering problems in a two-layer medium:

$$\Delta u + k^2 u = 0 \qquad \text{in } \mathbb{R}^2 \backslash \bar{D}, \tag{1.1a}$$

$$u = g \qquad \text{on } \Gamma_D, \tag{1.1b}$$

$$[u] = \left[\frac{\partial u}{\partial x_2}\right] = 0 \qquad \text{on } \Sigma, \tag{1.1c}$$

$$\sqrt{r}\left(\frac{\partial u}{\partial r} - \mathrm{i}ku\right) \to 0 \qquad \text{as } r = |x| \to \infty, \tag{1.1d}$$

where $D \subset \mathbb{R}^2$ is a bounded domain with Lipschitz boundary Γ_D and $g \in H^{1/2}(\Gamma_D)$, u is the scattering field, $\Sigma = \{(x_1,x_2) \in \mathbb{R}^2 : x_2 = 0\}$ is the interface, and $[u]_\Sigma := u_+ - u_-$ is the jump of u across Σ from above to below. We assume the wave number k is positive and piecewise constant, defined by

$$k(x) = \begin{cases} k_+, & \text{if } x \in \mathbb{R}^2_+, \\ k_-, & \text{if } x \in \mathbb{R}^2_-, \end{cases} \tag{1.2}$$

where $\mathbb{R}^2_\pm = \{(x_1,x_2) \in \mathbb{R}^2 : \pm x_2 > 0\}$. Without loss of generality we assume in this paper that $k_- > k_+ > 0$. We consider an acoustic incident wave in a two-layer medium. Due to the existence of Σ, the scattering waves consist of both propagating modes and evanescent modes. The problem geometry is shown in Fig. 1.

The PML method was first proposed by Bérénger [3] for solving the time dependent Maxwell equations. Following this, various constructions of PML absorbing layers have been proposed and studied in the literature. A detailed review of these methods can be found in Turkel and Yefet [39], Teixeira and Chew [38]. The basic idea of the PML method is to surround the computational domain by a layer of finite thickness with specially designed model medium that absorbs all the waves that propagate from inside the computational domain. Bao and Wu first proved the exponential convergence of the PML