

Developing Finite Element Methods for Simulating Transformation Optics Devices with Metamaterials

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Abstract. In this paper, we first develop the mathematical modeling equations for wave propagation in several transformation optics devices, including electromagnetic concentrator, rotator and splitter. Then we propose the corresponding finite element time-domain methods for simulating wave propagation in these transformation optics devices. We implement the proposed algorithms and our numerical results demonstrate the effectiveness of our modeling equations. To our best knowledge, this is the first work on time-domain finite element simulation carried out for the electromagnetic concentrator, rotator and splitter.

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1 Introduction

Due to its many interesting potential applications, electromagnetic (EM) metamaterials has been one of the hot research topics since year 2000 as evidenced by many recently published papers (e.g., [1, 13, 21, 26, 32, 44]) and monographs on metamaterials (e.g., [22, 29, 35, 37]). Metamaterials and transformation optics (TO) offer some new techniques to tame the electromagnetic fields to achieve the desired applications in engineering and materials sciences [8, 31]. In 2006, Pendry et al. [32] first presented the blueprints for realizing the EM cloak with metamaterials by using the TO theory. Since then, TO has attracted a great deal of attention [8, 10, 36, 37]. TO can be applied to design many

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interesting metamaterial devices, such as cloaking device, concentrator, rotator, and splitter, etc. In 2008, Jiang et al. [20] designed the arbitrarily shaped concentrators based on the conformally optical transformation. In the same time, Chen and Chan [9] realized the EM rotator and splitter by a layered structure of thin alternating layers of metal and dielectrics based on the TO theory.

In order to understand the electromagnetic wave behavior in various media, we need to solve the famous Maxwell's equations:

$$\begin{aligned}
 \text{Ampere's law: } & \frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}_s, \\
 \text{Faraday's law: } & \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \\
 \text{Gauss's laws: } & \nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0,
 \end{aligned} \tag{1.1}$$

where $\mathbf{E}, \mathbf{H}, \mathbf{D}, \mathbf{B}$ denote the electric field, magnetic field, electric flux density, magnetic flux density, respectively, and \mathbf{J}_s, ρ denote the electric current and electric charge density, respectively. To make the Maxwell's equations well-posed, we often need to couple the Maxwell's equations with the constitutive equations for complex media:

$$\mathbf{D} = \epsilon_0 \epsilon \mathbf{E}, \quad \mathbf{B} = \mu_0 \mu \mathbf{H}, \tag{1.2}$$

where ϵ_0 and μ_0 are the permittivity and permeability of vacuum, and ϵ and μ are the relative permittivity and permeability of the specific media.

There exist many excellent works on the analysis and applications of finite element method (FEM) for solving the Maxwell's equations in various media, such as in the free space (e.g., papers [3,4,6,7,11,16,17,43], books [12,22,30] and references therein); in general dispersive media (e.g., [2,19,28,34]); in negative index metamaterials [18,38,39]; and in cloaking metamaterials [23–25,41]. In recent years, we developed some mathematical models and time-domain FEMs for simulating the invisibility cloaks [23–25]. Continuing our previous efforts, here we develop some time-domain mathematical models and corresponding FEMs for simulating several transformation optics devices, including the EM concentrator [20,33,36,41], rotator and splitter [9,10]. Transformation optics has become a fantastic tool to design these devices with novel functionalities. Numerical simulation plays a very important role in helping engineers and physicists demonstrating those novel functionalities they expect and then constructing the real physical devices by using the material parameters tested in the simulation. However, except [41] with self developed spectral-element solver to simulate the electromagnetic concentrators and rotators, almost all simulations of EM concentrator, rotator and splitter are performed in the frequency domain by using COMSOL, a popular commercial software in metamaterial community. As broadband devices are proposed and realized (e.g., broadband ground-plane cloak [27], broadband light absorber [31] and broadband rotator [10]), the time-domain simulation becomes more important and needed. To the best of our knowledge, developing these mathematical models and solving them by the time-domain FEMs is original