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The Immersed Boundary Method: Application to Two-Phase Immiscible Flows

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Abstract. In this paper we present an extended formulation of the immersed boundary (IB) method that facilitates simulation of incompressible immiscible two-phase flows. In the developed formulation the pressure field and the surface tension forces associated with interface curvature are implicitly introduced in the form of distributed Lagrange multipliers. The approach provides for impermeability between both phases and exhibits accurate mass conservation without the need for additional correction procedures. Further, we present a grid independence study and extensive verification of the developed method for representative 2D two-phase flows dominated by buoyancy, shear stress, and surface tension forces.

AMS subject classifications: 76T10

Key words: Immersed boundary method, two-phase immiscible flow, distributed Lagrange multiplier.

1 Introduction

Numerical modeling of two-phase flow is a rapidly developing field, facilitating the solution of many problems both in applied science and in engineering. Numerical simulation of two-phase flows is used for the investigation of complex phenomena in the fields of environmental and geophysical science [1, 2], biomechanical engineering [3, 4], chemical processing [5] and the fabrication of optical wave guides. The simulation of two-phase flows is challenging, since it includes modeling of interactions between the different phases, tracking the boundary interface and, in some cases, resolving solidification and melting phase changes. A number of numerical techniques have been developed for modeling two-phase flows with deformable interfaces and free boundaries. Typically,

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the techniques are classified in terms of the multi-physics phenomena modeling: the flow modeling, the interface modeling, and the coupling between the two [6].

There are two basic numerical approaches that deal with free-interface two-phase flow: interface-tracking and interface-capturing. In the interface-tracking approach the interface is determined by a series of Lagrangian markers, whose location is dynamically updated throughout the numerical simulation. Front tracking (FT) [7, 8] and immersed boundary (IB) [9–12] methods are typical examples of the interface tracking approach. In the interface-capturing approach the interface is reconstructed from an Eulerian scalar field characterizing the flow properties. This approach includes the volume of fluid (VOF) [8, 13], level set (LS) [8, 14] and phase field (PF) methods [8, 15, 16].

The present study focuses on the development of a novel formulation of the interface-tracking approach based on the direct forcing IB method [17, 18]. The IB method was initially introduced by Peskin [9, 10] for simulation of blood dynamics in the cardiac chambers. The method is suitable for the simulation of flows in the presence of a number of immersed bodies of arbitrary geometry. Each body is determined by a set of Lagrangian points which do not necessarily coincide with the underlying Eulerian grid. In the most general case the body can be deformable and moving. In the direct forcing IB approach the kinematic non-slip constraints at all the points of the body are enforced by applying Lagrangian forces entering as sources into the Navier Stokes (NS) equations. The values of the applied forces are unknown a priori and are a part of the overall solution of the problem. In single-phase flows, the values of the Lagrangian forces are coupled with the pressure and velocity fields governed by the NS equations. In immiscible two-phase flows, the simulation should also account for the surface tension forces coupling the fluid characteristics of each phase with the unknown dynamically evolving curvature of the interphase interface.

The accuracy of any two-phase numerical simulation employing the interface tracking approach depends on a precise evaluation of the Lagrangian forces on the interface between the two phases. The forces comprise the kinematic constraints for continuous values of shear stress and velocity vectors. Historically, numerical simulations relied on explicit treatment of Lagrangian forces (see e.g. Li et al. [12], Rutka and Li [19]). The surface tension forces are explicitly calculated based on the interface curvature obtained at the previous time step, while the NS equations are solved by the SIMPLE [20] algorithm. The above methodology can be easily plugged into any existing time marching solver of the NS equations based on a segregated pressure-velocity coupling, which explains its high popularity for simulation of both single- and two-phase flows [17, 18, 21, 22]. However, the explicit scheme has a number of disadvantages. First, the kinematic constraints are applied to the intermediate velocity field, which has to be further projected to a divergence free subspace. As a result, a non-negligible mass leakage through the interface between the two phases typically shows up after completing the correction-projection step. To improve the accuracy and to ensure the mass conservation of the explicit direct forcing IB formulation a number of techniques have been proposed. Worthy of note are the works of Kempe et al. [23, 24], who imposed substantially more accurate boundary