Theory of Spontaneous Symmetry Breaking and an Application to Superconductivity: Nambu-Goldstone and Higgs Excitation Modes

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Abstract. We present a general framework of the theory of spontaneous symmetry breaking in non-relativistic systems. We discuss a spontaneous symmetry breaking in a system with general global symmetry given by a Lie group $G$. The Nambu-Goldstone boson and Higgs boson are represented explicitly by local fields by means of the basis of the Lie algebra of $G$. An application to superconductivity is discussed. We evaluate the Green’s functions of the Nambu-Goldstone and Higgs bosons in superconductors. We show that the Nambu-Goldstone and Higgs modes exhibit interesting behaviors in multi-component superconductors.

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1 Introduction

When the Lagrangian is invariant under a symmetry transformation, there is a conserved current and a conserved quantity. The spontaneous symmetry breaking indicates that the state is not invariant under symmetry transformation although the Lagrangian is invariant under this transformation. This occurs when an asymmetric state is realized in a symmetric system. It is well known that when a continuous symmetry is spontaneously broken, a massless boson appears. This boson is called the Nambu-Goldstone boson (NG boson) [1–3]. General proofs of the existence of the NG boson were given in [3, 4]. The spontaneous symmetry breaking has been studied intensively in the condensed-matter physics [5–10] and in field theory [11–21].

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We investigate a general formulation of the Nambu-Goldstone boson in a system with spontaneous symmetry breaking. The model has a symmetry of continuous Lie group $G$. The Nambu-Goldstone boson is represented by means of local fields with bases of Lie algebra of $G$. The existence of the massless boson is shown on the basis of this representation. The Ward-Takahashi identity is modified to include a vertex correction due to the Nambu-Goldstone boson in a spontaneous symmetry broken system. This means that the breaking of the Ward-Takahashi identity owing to spontaneous symmetry breaking is compensated by the inclusion of the Nambu-Goldstone boson. We also discuss applications to physical systems. In particular, we examine the Nambu-Goldstone and Higgs modes in superconductors.

This paper is organized as follows. In Section II, we investigate spontaneous symmetry breaking for a general compact group $G$. We discuss the Ward-Takahashi identity in this section. In Section III, we discuss excitation modes in superconductors. The Section IV is devoted to an investigation of the low energy Nambu-Goldstone and Higgs modes in multi-band superconductors. We give a summary in last section.

2 Spontaneous symmetry breaking

2.1 Symmetry Group

Let $G$ be a compact Lie group (symmetry group) and $g$ be the Lie algebra of $G$. The elements of the basis set of the Lie algebra $g$ as $T_a (a=1,\cdots,N_G)$ where $N_G$ is the dimension of $G$. We adopt that the fermion field $\psi$ is transformed under the action of the Lie group $G$ as

$$\psi \rightarrow e^{-i\theta T_a}\psi = \psi - i\theta T_a \psi + \mathcal{O}(\theta^2),$$

(2.1)

where $\theta$ is an infinitesimal parameter. We put $\delta \psi = -i\theta T_a \psi$. $\{T_a\}$ are normalized as

$$\text{Tr}T_a T_b = c \delta_{ab},$$

(2.2)

where $c$ is a real constant. The structure constants are introduced through commutation relations,

$$[T_a, T_b] = \sum_c i f_{abc} T_c.$$  

(2.3)

When the Lagrangian $\mathcal{L}$ is invariant under the transformation $\psi \rightarrow \psi + \delta \psi$, there is a conserved current. We denote the current for the transformation generated by $T_a$ as $j^\mu_a$:

$$j^\mu_a = \frac{\delta \mathcal{L}}{\delta (\partial^\mu \psi)} \delta \psi.$$  

(2.4)

This satisfies $\partial_\mu j^\mu_a = 0$. The conserved quantities are defined by

$$Q_a = \int d^4r j^0_a (r),$$

(2.5)