Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2016-0179c

Vol. **23**, No. 2, pp. 315-360
February 2018

Dispersive Shallow Water Wave Modelling. Part III: Model Derivation on a Globally Spherical Geometry

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Received 23 October 2016; Accepted (in revised version) 26 June 2017

Abstract. The present article is the third part of a series of papers devoted to the shallow water wave modelling. In this part we investigate the derivation of some long wave models on a deformed sphere. We propose first a suitable for our purposes formulation of the full EULER equations on a sphere. Then, by applying the depth-averaging procedure we derive first a new fully nonlinear weakly dispersive base model. After this step we show how to obtain some weakly nonlinear models on the sphere in the so-called BOUSSINESQ regime. We have to say that the proposed base model contains an additional velocity variable which has to be specified by a closure relation. Physically, it represents a dispersive correction to the velocity vector. So, the main outcome of our article should be rather considered as a whole family of long wave models.

AMS subject classifications: 76B15, 76B25

PACS: 47.35.Bb, 47.35.Fg

Key words: Motion on a sphere, long wave approximation, nonlinear dispersive waves, spherical geometry, flow on sphere.

1 Introduction

Recent mega-tsunami events in Sumatra 2004 [1,47,69] and in Tohoku, Japan 2011 [31,57] required the simulation of tsunami wave propagation on the global trans-oceanic scale. Moreover, similar catastrophic events in the future are to be expected in these regions [54]. The potential tsunami hazard caused by various seismic scenarii can be estimated by extensive numerical simulations. During recent years the modelling challenges of

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tsunami waves have been extensively discussed [14, 59]. On such scales the effects of Earth's rotation and geometry might become important. Several authors arrived to this conclusion, see *e.g.* [15,32]. There is an intermediate stage where the model is written on a tangent plane to the sphere in a well-chosen point. In the present study we consider the globally spherical geometry without such local simplifications.

The direct application of full hydrodynamic models such as EULER or NAVIER–STOKES equations does not seem realistic nowadays. Consequently, approximate mathematical models for free surface hydrodynamics on rotating spherical geometries have to be proposed. This is the main goal of the present study. The existing (dispersive and non-dispersive) shallow water wave models on a sphere will be reviewed below. Nowadays, hydrostatic models are mostly used on a sphere [73, 79]. The importance of frequency dispersion effects was underlined in *e.g.* [71]. Their importance has been realized for tsunami waves generated by sliding/falling masses [4, 22, 23, 76]. However, we believe that on global trans-oceanic scales frequency dispersion effects might have enough time to accumulate and, hence, to play a certain rôle. Finally, the topic of numerical simulation of these equations on a sphere is another important practical issue. It will be addressed in some detail in the following (and the last) Part IV [38] of the present series of papers entirely devoted to shallow water wave modelling.

Shallow water equations describing long wave dynamics on a (rotating) sphere have been routinely used in the fields of Meteorology and Climatology [79]. Indeed, there exist many similarities in the construction of approximate models of atmosphere and ocean dynamics [53]. The derivation of these equations by depth-averaging can be found in the classical monograph [33]. The main numerical difficulties here consist mainly in (structured) mesh generation on a sphere and treating the degeneration of governing equations at poles (the so-called poles problem). So far, the finite differences [46, 49] and spectral methods [9] were the most successful in the numerical solution of these equations. Our approach to these problems will be described in [38].

It is difficult to say who was the first to apply Nonlinear Shallow Water Equations (NSWE) on a sphere to the problems of Hydrodynamics. Contrary to the Meteorology, where the scales are planetary from the outset and the spherical coordinates are introduced even on local scales [52], in surface wave dynamics people historically tended to use local CARTESIAN coordinates. However, the need to simulate trans-oceanic tsunami wave propagation obliges us to consider spherical and Earth's rotation effects. We would like to mention that in numerical modelling of water waves on the planetary scale the problem of poles does not arise since these regions are covered with the ice. Thus, the flow cannot take place there.

In [77] one can find various forms of shallow water equations on a sphere along with standard test cases to validate numerical algorithms. The standard form of Nonlinear Shallow Water Equations (NSWE) in the spherical coordinates $O\lambda\varphi r$ is

$$\mathcal{H}_t + \boldsymbol{\nabla}\cdot[\mathcal{H}\boldsymbol{u}] = 0,$$