A Novel Multi-Dimensional Limiter for High-Order Finite Volume Methods on Unstructured Grids

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Abstract. This paper proposes a novel distance derivative weighted ENO (DDWENO) limiter based on fixed reconstruction stencil and applies it to the second- and high-order finite volume method on unstructured grids. We choose the standard deviation coefficients of the flow variables as the smooth indicators by using the k-exact reconstruction method, and obtain the limited derivatives of the flow variables by weighting all derivatives of each cell according to smoothness. Furthermore, an additional weighting coefficient related to distance is introduced to emphasize the contribution of the central cell in smooth regions. The developed limiter, combining the advantages of the slope limiters and WENO-type limiters, can achieve the similar effect of WENO schemes in the fixed stencil with high computational efficiency. The numerical cases demonstrate that the DDWENO limiter can preserve the numerical accuracy in smooth regions, and capture the shock waves clearly and steeply as well.

1 Introduction

Computational fluid dynamics (CFD) researchers have kept pursuing the accuracy of numerical simulation and its adaptability to complex configurations. Although the second-order accuracy schemes have played an important role in aircraft design and application [1–4], they still have large numerical dissipation and dispersion. For some complex

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flow problems, such as wave propagation, vortex-dominated flows including high-lift configuration, helicopter blade vortex interaction, as well as large eddy simulation and direct numerical simulation of turbulence, high-order accuracy schemes must be used to obtain more elaborate and detailed results [5]. However, whether for the second- or the high-order scheme, an outstanding issue of solving discontinuous flowfields is how to clearly and precisely capture shock waves and suppress non-physical oscillations near discontinuities. It is technically difficult to develop higher-order schemes on unstructured grids, as they tend to have lower dissipation and weaker robustness compared with second-order schemes [6, 7]. Therefore, an efficient and accurate oscillation control strategy should be incorporated into both the second- and the high-order numerical schemes.

Limiting the numerical process is one of the most commonly used ways to deal with discontinuities, such as shock waves. Currently, there are mainly three types of limiters. The first one is using artificial viscosity [8–10]. This limiter can smooth numerical oscillations near shock waves by introducing artificial dissipative terms into flow control equations. It is robust and easy to implement. However, the value of the additional terms needs to be modulated by free parameters. Besides, the artificial viscosity is a kind of numerical viscosity, which may affect the real physical viscosity to a certain extent, especially for viscous flows.

The second limiter utilizes ENO/WENO schemes [11–24]. The ENO schemes, first proposed by Harten [11, 12], have been successfully applied in flowfield with strong shock waves. The smoothest stencil is selected to reconstruct the distribution of flow variables; while other stencils containing discontinuities are all discarded. Unlike ENO, the WENO schemes [13–17] combine all of the stencils by assigning a nonlinear weight coefficient to each reconstruction stencil based on the local smoothness of flowfield. Although the WENO schemes show superior advantages in capturing shocks and have been successfully applied, it is a enormous task to choose admissible and proper stencils from a large number of cells, especially for high-order and multi-dimensional cases on unstructured grids. Qiu and Shu [18] proposed the Hermite WENO (HWENO) scheme and used it as a limiter for Runge-Kutta discontinuous Galerkin method. In comparison with traditional WENO schemes, the main difference of the HWENO scheme is that both the function and its first derivatives are evolved in time and used in reconstruction. Therefore, the scheme has a more compact stencil for the same order of accuracy, and it is widely used on unstructured grids to control spurious oscillations, especially for discontinuous Galerkin method [19]. However, both the flow variables and their derivatives of each cell are evolved in time marching, and more storage and CPU time are required. Rather than choosing candidate stencils, Ivan and Groth [20] proposed a high-order central ENO (CENO) finite volume scheme for solving compressible flows on unstructured grids. The method is performed on fixed central stencils, which involves an unlimited k-exact reconstruction in smooth regions, and switches a limited piecewise linear reconstruction when discontinuities are identified. The CENO scheme is also extended to three-dimensional cases, in which the robustness and the high-order accuracy