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A Cartesian Scheme for Compressible Multimaterial Hyperelastic Models with Plasticity

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Abstract. We describe a numerical model to simulate the non-linear elasto-plastic dynamics of compressible materials. The model is fully Eulerian and it is discretized on a fixed Cartesian mesh. The hyperelastic constitutive law considered is neohookean and the plasticity model is based on a multiplicative decomposition of the inverse deformation tensor. The model is thermodynamically consistent and it is shown to be stable in the sense that the norm of the deviatoric stress tensor beyond yield is non increasing. The multimaterial integration scheme is based on a simple numerical flux function that keeps the interfaces sharp. Numerical illustrations in one to three space dimensions of high-speed multimaterial impacts in air are presented.

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1 Introduction

The numerical modelling of multimaterial rapid dynamics in extreme conditions is an important technological problem for industrial and scientific applications. Experiments are dangerous, need heavy infrastructures and hence are difficult and expensive to realize. The simulation of such phenomena is challenging because they couple large deformations and displacements in solids to strongly non-linear behaviour in fluids. In what follows, we privilege a fully Eulerian approach based on conservation laws, where the different materials are characterized by their specific constitutive laws. This approach was

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introduced in [10] and subsequently pursued and extended for example in [2, 9, 13, 19]. For specific applications, a Lagrangian approach like in [17] or [15] can be more suitable.

In this work we extend to elasto-plastic flows the schemes presented in [5, 11] for hyperelastic multimaterials. This scheme is based on a simple modification of the numerical flux function at the multimaterial interface that allows an efficient code parallelization. Thanks to this scheme, there is no need of defining a ghost fluid through the material boundaries to avoid accuracy and stability issues. In addition, this interface remains sharp.

Plasticity modelling is an open issue. The models are phenomenological and several problematic points still need further investigation, see for example [20]. Here we follow an established literature [1, 6, 12, 16, 18, 21, 22] where the deformation is viewed as the composition of a purely elastic and a purely plastic mapping. This approach has the advantage that plastic effects are modeled as a source term in the equation for the elastic deformation tensor. Also, using appropriate constitutive equations for the plastic phenomenon, it is possible to respect thermodynamic consistency (entropy is increasing) and phenomenological constraints as volume invariance of the plastic flow. In particular, we show here that with the plasticity model adopted in [18], the deviatoric stress tensor norm is actually non increasing during the plastic process for the neohooken hyperelastic model.

The applications we present are illustrations of the stiff phenomena occurring when high speed projectiles impact on shields. These examples include a one-dimensional case where we can compare the numerical results to an exact solution, a two-dimensional impact on a plastic layer, two and three-dimensional impacts on framed plastic shields.

2 Eulerian hyperelastic model

This model was already discussed in [3, 8, 10, 19, 23, 24]. We follow here the formulation presented in [5] and extend it to plasticity modelling. The equations of mass, momentum, deformation and energy conservation are given by

$$\begin{cases} \partial_t \rho + \mathrm{div}_x(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}_x(\rho u \otimes u - \sigma) = 0, \\ \partial_t(\nabla_x Y) + \nabla_x(u \cdot \nabla_x Y) = 0, \\ \partial_t(\rho e) + \mathrm{div}_x(\rho e u - \sigma^T u) = 0. \end{cases} \tag{2.1}$$

The physical variables are the density $\rho(x,t)$, the velocity $u(x,t)$, the total energy per unit mass $e(x,t)$ and the Cauchy stress tensor in the physical domain $\sigma(x,t)$. Here $Y(x,t)$ is the backward characteristics that for a time t and a point x in the deformed configuration, gives the corresponding initial point in the initial configuration. The equation on $\nabla_x Y$ is required in order to record the deformation in the Eulerian frame. The initial density $\rho(x,0)$, the initial velocity $u(x,0)$, the initial total energy $e(x,0)$ and $\nabla_x Y(x,0) = I$ are given together with appropriate boundary conditions.