COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **1**, No. 4, pp. 744-764

Commun. Comput. Phys.
August 2006

A Fourth-Order Upwinding Embedded Boundary Method (UEBM) for Maxwell's Equations in Media with Material Interfaces: Part I

Shaozhong Deng and Wei Cai*

Department of Mathematics and Statistics, University of North Carolina at Charlotte, Charlotte, NC 28223-0001, USA.

Received 1 November, 2005; Accepted (in revised version) 10 February, 2006

Abstract. In this paper, we present a new fourth-order upwinding embedded boundary method (UEBM) over Cartesian grids, originally proposed in the *Journal of Computational Physics [190 (2003), pp. 159-183.]* as a second-order method for treating material interfaces for Maxwell's equations. In addition to the idea of the UEBM to evolve solutions at interfaces, we utilize the ghost fluid method to construct finite difference approximation of spatial derivatives at Cartesian grid points near the material interfaces. As a result, Runge-Kutta type time discretization can be used for the semi-discretized system to yield an overall fourth-order method, in contrast to the original second-order UEBM based on a Lax-Wendroff type difference. The final scheme allows time step sizes independent of the interface locations. Numerical examples are given to demonstrate the fourth-order accuracy as well as the stability of the method. We tested the scheme for several wave problems with various material interface locations, including electromagnetic scattering of a plane wave incident on a planar boundary and a two-dimensional electromagnetic application with an interface parallel to the y-axis.

Key words: Yee scheme; upwinding embedded boundary method (UEBM); ghost fluid method (GFM); Maxwell's equations.

1 Introduction

The finite difference time domain (FDTD) Yee scheme, first introduced by Yee in 1966 [1] and later developed by Taflove and others [2], has been used for a broad range of application problems in computational electromagnetics. The staggered Yee scheme has

*Correspondence to: Wei Cai, Department of Mathematics and Statistics, University of North Carolina at Charlotte, Charlotte, NC 28223-0001, USA. Email: `wcai@uncc.edu`

been demonstrated to be robust, efficient, and simple to implement. However, when used to model curved objects or to solve Maxwell's equations in media with material interfaces, the Yee scheme requires locally conforming meshes for the irregular boundaries or the material interfaces. Otherwise, it will reduce to at best first-order accuracy and may produce locally non-convergent results [3, 4]. Furthermore, for Maxwell's equations with discontinuous coefficients, the Yee scheme might not be able to capture the possible discontinuity of the solution across the interfaces.

A number of finite difference methods have been proposed in the past for modeling time-domain Maxwell's equations with curved material interfaces. The usual and straightforward approach is to introduce appropriate local modifications into the Yee scheme but still keep the staggered grid [4, 5]. Recently, there are some studies of high-order embedded FDTD schemes for time-domain Maxwell's equations with material interfaces, including the non-dissipative staggered fourth-order accurate explicit method and the staggered fourth-order compact implicit method by Yefet et al [6, 7], and the explicit fourth-order staggered method and the explicit fourth-order orthogonal curvilinear staggered grid method by Xie et al [8, 9]. Also, high-order FDTD methods via hierarchical implicit derivative matching are presented in [10].

In this paper, we present a new fourth-order upwinding embedded boundary method (UEBM) over Cartesian grids, originally proposed in [11] as a second-order method for treating material interfaces for Maxwell's equations. In addition to the idea of the UEBM to evolve solutions at the interfaces, we utilize the ghost fluid method to construct finite difference approximation of spatial derivatives at Cartesian grid points near the material interfaces. As a result, Runge-Kutta type time discretization can be used for the semi-discretized system to yield an overall fourth-order method, in contrast to the original second-order method based on a Lax-Wendroff type difference. The fourth-order method still uses a simple Cartesian grid and a central difference scheme for mesh points away from the interfaces. Solutions at both sides of the interfaces are calculated with an upwinding strategy while preserving the possible physical jump conditions. Previous numerical methods making use of Cartesian grids for the approximation of one-dimensional hyperbolic equations could also be found in [12–14].

The ghost fluid method (GFM) was originally designed to treat contact discontinuities in the inviscid Euler equations in [15], and since then it has been generalized to handle irregular boundaries in a variety of problems [16–28]. For examples, with the use of the so-called ghost cells (based on the GFM), Gibou et al proposed in [20] a second-order accurate finite difference method for Poisson equations, and most recently in [21] a fourth-order accurate finite difference discretization for the Laplace and heat equations on irregular domains with Dirichlet boundary conditions on the irregular interfaces. For second-order wave equations, by using ghost points on either side of the interfaces, Kreiss et al proposed several second-order embedded boundary methods with Dirichlet boundary condition [25, 26], Neumann boundary condition [27], and jump conditions [28] on the irregular interfaces, respectively.

In this paper, we shall combine the GFM with the UEBM to derive high-order Carte-