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# An All-Regime Lagrange-Projection Like Scheme for the Gas Dynamics Equations on Unstructured Meshes

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**Abstract.** We propose an *all regime* Lagrange-Projection like numerical scheme for the gas dynamics equations. By *all regime*, we mean that the numerical scheme is able to compute accurate approximate solutions with an under-resolved discretization with respect to the Mach number M, i.e. such that the ratio between the Mach number M and the mesh size or the time step is small with respect to 1. The key idea is to decouple acoustic and transport phenomenon and then alter the numerical flux in the acoustic approximation to obtain a uniform truncation error in term of $M$. This modified scheme is conservative and endowed with good stability properties with respect to the positivity of the density and the internal energy. A discrete entropy inequality under a condition on the modification is obtained thanks to a reinterpretation of the modified scheme in the Harten Lax and van Leer formalism. A natural extension to multi-dimensional problems discretized over unstructured mesh is proposed. Then a simple and efficient semi implicit scheme is also proposed. The resulting scheme is stable under a CFL condition driven by the (slow) material waves and not by the (fast) acoustic waves and so verifies the *all regime* property. Numerical evidences are proposed and show the ability of the scheme to deal with tests where the flow regime may vary from low to high Mach values.

**AMS subject classifications**: 35L65, 35L40, 65M08, 76N15, 76M12, 76B99

**Key words**: Gas dynamics equations, low-Mach regime, finite volume schemes, all-regime schemes, Lagrange-Projection like schemes, large time-steps.

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## 1 Introduction

In this paper, we consider the system of gas dynamics in two space dimension in situations when the flow regime may vary in terms of Mach number $M$ across the computational domain. We propose a collocated Finite Volume method that addresses two important issues.

The first issue concerns the lack of accuracy in the low Mach regime of Godunov-type schemes. While these methods perform well at capturing shocks, they may generate spurious numerical diffusion when they are used for simulating low Mach flows over relatively coarse mesh with respect to the Mach number. Improvements of Godunov-type schemes more generally of collocated methods have been proposed by many authors like [5,7–9,11,15,19,21,22,24,26,27,32,33]. The analysis of these authors may rely on different arguments like the analysis of the viscosity matrix [33], an asymptotic expansion in terms of Mach number [19], a detailed study in [11] that seek for invariance properties of the numerical scheme transposing the framework of Schochet [28] to the discrete setting, and also an analysis based on the so-called Asymptotic Preserving property [23] in [22]. Nevertheless the resulting cure usually boils down to reduce the numerical diffusion in the momentum equation for low Mach number values.

The second problem we address deals with subsonic flow when the fluid velocity is slow and the acoustic waves are not driving phenomenons. In this case, the Courant-Friedrichs-Lewy (CFL) condition on the time step for explicit Godunov-type methods that involves the (fast) acoustic wave velocity may lead to very small time steps choices and thus costly computations. It seems natural to seek for numerical schemes that enable the use of a large time steps that are not constrained by the sound velocity. This question has been examined by several authors like [7–9,22,24] (see also [4,6]) who derived mixed implicit-explicit strategies that allows to choose the time step independently of the Mach Number.

Numerical schemes that can tackle both issues, namely: accuracy for mesh sizes that do not depend on the Mach number and also stability for time steps that are not constrained by the Mach value are usually referred to as *all regime*, like the methods proposed by [7–9,22,24].

In the present work, we first propose an operator splitting strategy that allows to decouple the acoustic and the transport phenomenons. The approximation algorithm is split into two steps: an acoustic step and a transport step. For one-dimensional problems, this strategy is equivalent to an explicit Lagrange-Projection [14,18] method, however the present splitting does not involve any moving Lagrangian mesh and can be naturally expressed for multi-dimensional problems. Following simple lines inspired by [10,11] we investigate the dependence of the truncation error with respect to the Mach number. Let us mention that our study does not involve a Taylor expansion in the vicinity of the zero-Mach limit, nor a near-divergence free condition for the velocity field. Although this analysis is by no mean a thorough explanation of the low Mach regime behavior of our solver, it is enough to suggest simple means to obtain a truncation error with a uniform