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Theoretical and Numerical Modeling of Nonlinear Electromechanics with applications to Biological Active Media

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Abstract. We present a general theoretical framework for the formulation of the nonlinear electromechanics of polymeric and biological active media. The approach developed here is based on the additive decomposition of the Helmholtz free energy in elastic and inelastic parts and on the multiplicative decomposition of the deformation gradient in passive and active parts. We describe a thermodynamically sound scenario that accounts for geometric and material nonlinearities. In view of numerical applications, we specialize the general approach to a particular material model accounting for the behavior of fiber reinforced tissues. Specifically, we use the model to solve via finite elements a uniaxial electromechanical problem dynamically activated by an electrophysiological stimulus. Implications for nonlinear solid mechanics and computational electrophysiology are finally discussed.

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Key words: Active electromechanical media, Helmholtz free energy, multiplicative decomposition of the deformation gradient, active deformation, active stress.

1 Introduction

Electro-elastic (EA) media are physical systems that are sensitive to the action of mechanical forces and electric fields. When immersed in electric fields, EA systems deform spontaneously, and, when deformed by mechanical forces, they cause a change in the original

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configuration of electrostatic or electrodynamic fields. The variation of the assigned configuration of the electric field lines triggered by the electromechanical coupling is called mechanic-electric feedback (MEF). Typically, in EA systems deformations may induce a change of the eventual initial isotropy of a body.

Historically, the most well known example of electro-elastic systems has been the piezoelectric crystal. In linearized kinematics, it can be proved that an isotropic dielectric immersed in an electric field develops polarization charges, inducing internal stresses proportional to the square of the electric field [37]. A similar dynamics characterizes piezoelectrics. The origin of the electromechanical coupling in piezoelectric materials stems from a phase transition that breaks the symmetry, and that leads also to a spontaneous polarization. By this spontaneous polarization there is a linear coupling between deformation and electric field [36, 44], so that MEF effects are enhanced. Piezoelectrics manifest also the reverse feedback: imposed deformations induce an internal electric field proportional to the magnitude of the deformations. A second important class of materials where MEF is of relevance are electro-active polymers (EAP), that typically exhibit changes in size or in shape when stimulated by an electric field [53]. Among the recent literature addressing this class of materials, it is worth to mention contributions concerning electro-visco elastic polymers [4, 5, 74], and proposing thermodynamic formulations for electro-active synthetic materials [46].

The MEF effect is observed also in materials that are in focus in the present study, i.e., biological media with contractile properties, such as the heart, the intestines, and several types of muscles. It is evident that biological systems undergo rather large deformations, therefore the underlying biophysical dynamics cannot be described accurately by means of the infinitesimal theory of elasticity. In particular, according to single cell and tissue specimen measurements, during a normal heart beat myocytes change their length up to 20% [50], i.e., in the typical range of finite deformations. According to the literature, the mechanical properties of muscles have been mainly investigated at the macroscopic scale via force-velocity relationships, following Hill's model [30]. The complexity of the electric fields and of the mechanics of the heart, however, requires to adopt a multiscale perspective, since the cardiac contraction connects the global mechanical properties observed at the organ scale [78] to the underlying subcellular dynamics [32].

The cardiac beating is the result of the propagation of electrical waves generated by the sequential excitation of neighboring cells, located along specialized conductive structures that provide the spreading of the electric signal into the whole heart [35, 49, 57, 66]. In turn, the excitation of a cardiac cell is induced by the variation of the electric potential across the cell membrane. Changes in the electric potential are related in a nonlinear manner to the transmembrane fluxes of various charged ions.

The basic features of the mechanical response of biological active tissues can be sufficiently well described by hyperelastic models, disregarding in first approximation more complicated effects related to viscosity, growth and remodeling. Since the '80s, several mathematical models of passive muscle and myocardium elasticity have been proposed, including isotropic, transversely isotropic and, more recently, orthotropic models [31, 39].