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Thin Layer Models for Electromagnetism

Marc Duruflé[1], Victor Péron[2] and Clair Poignard[3,*]

[1] *Team Magique-3D, INRIA Bordeaux-Sud-Ouest, Institut de Mathématiques de Bordeaux, CNRS UMR 5251 & Université de Bordeaux1, 351 cours de la Libération, 33405 Talence Cedex, France.*
[2] *Team Magique-3D, INRIA Bordeaux-Sud-Ouest, LMAP CNRS UMR 5142 & Université de Pau et des Pays de l'Adour, France.*
[3] *Team MC2, INRIA Bordeaux-Sud-Ouest, Institut de Mathématiques de Bordeaux, CNRS UMR 5251 & Université de Bordeaux1, 351 cours de la Libération, 33405 Talence Cedex, France.*

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Abstract. We present a review on the accuracy of asymptotic models for the scattering problem of electromagnetic waves in domains with thin layer. These models appear as first order approximations of the electromagnetic field. They are obtained thanks to a multiscale expansion of the exact solution with respect to the thickness of the thin layer, that makes possible to replace the thin layer by approximate conditions. We present the advantages and the drawbacks of several approximations together with numerical validations and simulations. The main motivation of this work concerns the computation of electromagnetic field in biological cells. The main difficulty to compute the local electric field lies in the thinness of the membrane and in the high contrast between the electrical conductivities of the cytoplasm and of the membrane, which provides a specific behavior of the electromagnetic field at low frequencies.

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1 Introduction

The aim of this work is to provide a review of several asymptotic models for the scattering problem of time-harmonic electromagnetic waves in domains with thin layer. Media with

*Corresponding author. *Email addresses:* marc.durufle@inria.fr (M. Duruflé), victor.peron@inria.fr (V. Péron), clair.poignard@inria.fr (C. Poignard)

thin inclusions appear in many domains: geophysical applications, microwave imaging, biomedical applications, cell phone radiations, radar applications, non-destructive testing$\cdots$. In this paper, the simplified configuration is mainly motivated by the computation of the electromagnetic field in biological cells.

The electromagnetic modeling of biological cells has become extremely important since several years, in particular in the biomedical research area. In the simple model of Fear, and Stuchly or Foster and Schwan [8–10], the biological cell is composed of a conducting cytoplasm surrounded by a thin insulating membrane. When the cell is exposed to an electric field, the local field near the membrane may overcome physiological values. Then, complex phenomenon known as electropermeabilization (or electroporation) may occur [21]: the cell membrane is destructured and some outer molecules might be internalized inside the cell, as described in the model of Kavian *et al.* [11]. These process hold great promises in oncology and gene therapy, particularly, to deliver drug molecules in cancer treatment. This is the reason why several papers in the bioelectromagnetic research area deal with numerical modeling of the cell (see for instance [12, 19, 20]) and with numerical computations of the membrane voltage. Actually, the main difficulties of the calculation of the local electric field lie in the thinness of the membrane and in the high contrast between the electromagnetic properties of the cytoplasm and the membrane. More precisely, though the electric permittivities of these two media are of the same order of magnitude, the membrane conductivity is much lower than the cytoplasm conductivity, which provides particular behavior of the electromagnetic field at low frequencies, for which the conduction currents predominate.

In previous papers [15–18], Poignard *et al.* have proposed an asymptotic analysis to compute the solution to the conductivity problem, the so-called electric potential, in domains with thin layer. In particular, Perrussel and Poignard have derived the asymptotic expansion of the electric potential at any order in domains with resistive thin layer [14]. More recently, we have derived an asymptotic model for the solution to time-harmonic Maxwell equations, the so-called electromagnetic field, in biological cell at mid-frequency [7, Eq. (5.1)]. In the proceeding [6, Sec. 3], we have derived an asymptotic model for the electromagnetic field in biological cell at low-frequency which corresponds to a resistive membrane. All these papers are based on a multi-scale asymptotic expansion of the partial differential equations, that makes possible to replace the thin layer by appropriate transmission conditions.

In [7], we have derived a multi-scale expansion for the electric field in power series of a small parameter ε, which represents the relative size of the cell membrane [7, Eq. (5.1)]. We inferred appropriate transmission conditions "at first order" on the boundary of the cytoplasm satisfied by the first two terms of the expansion $\mathbf{E}^0+\varepsilon\mathbf{E}^1$. We proved uniform estimates (in energy norm) with respect to ε for the error between the exact solution $\mathbf{E}^\varepsilon$ and the approximate solution $\mathbf{E}^0+\varepsilon\mathbf{E}^1$ [7, Th. 2.9]. We validated the asymptotic expansion up to the first two terms, proving estimates for the remainder of the expansion defined as $\mathbf{E}^\varepsilon-(\mathbf{E}^0+\varepsilon\mathbf{E}^1)$ [7, Th. 6.3]. We recall in Sec. 3.1 the two first order of this asymptotic expansion which is relevant in the mid-frequency range. This expansion is no longer