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Radar Cross Section Reduction of a Cavity in the Ground Plane

Gang Bao[1,2,*] and Jun Lai[2]

[1] *Department of Mathematics, Zhejiang University, Hangzhou 310027, Zhejiang, China.*
[2] *Department of Mathematics, Michigan State University, East Lansing, MI 48824, USA.*

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Abstract. This paper investigates the reduction of backscatter radar cross section (RCS) for a rectangular cavity embedded in the ground plane. The bottom of the cavity is coated by a thin, multilayered radar absorbing material (RAM) with possibly different permittivities. The objective is to minimize the backscatter RCS by the incidence of a plane wave over a single or a set of incident angles. By formulating the scattering problem as a Helmholtz equation with artificial boundary condition, the gradient with respect to the material permittivities is determined efficiently by the adjoint state method, which is integrated into a nonlinear optimization scheme. Numerical example shows the RCS may be significantly reduced.

AMS subject classifications: 35Q93, 35J05

Key words: Optimal design, RCS reduction, adjoint method, radar absorbing materials.

1 Introduction

Radar cross section (RCS) is an important measure for the detection of a target by radar systems. The RCS from a cavity is significant since the overall RCS of a target is often dominated by some cavities, such as the jet inlet of an aircraft. Therefore, effective reduction of the RCS from a cavity has been an important problem in wave propagation with many practical applications. In this paper, we focus on a 2-D rectangular cavity which is embedded in the ground plane and illuminated by a time-harmonic plane wave. A thin, multilayered RAM is coated horizontally at the bottom of the cavity for the reduction of backscatter RCS, as shown in Fig. 1.

*Corresponding author. *Email addresses:* bao@math.msu.edu (G. Bao), laijun@msu.edu (J. Lai)

Many methods have been proposed on RCS reduction with RAM [7,15,17]. A popular one is based on genetic algorithm (GA), which is a gradient-free optimization method and widely used in engineering community. The problem is, even for a small parameter space, GA usually requires thousands of generations to obtain the best population, or the global optimal. While designing a fast algorithm for the direct scattering problem already presents a huge challenge especially when the cavity is large and deep [10,13], it is computationally unaffordable to employ GA to find the optimal synthesized RAM of a cavity.

In this paper, the problem is addressed by the gradient-based sequential quadratic programming (SQP) method. SQP is a one of the most effective optimization methods particularly for nonlinear constraint like partial differential equations (PDE) [16]. The basic idea of this method is to generate steps by solving a sequence of quadratic subproblems. The gradient involved in the quadratic subproblems is provided accurately by the continuous adjoint state method. The accuracy plays an important role in the design especially when the backscatter RCS is highly sensitive to the change of permittivities.

Due to the difficulty in the forward scattering problem of a cavity as we mentioned, a crucial step underlying the optimization method is to find a fast solver for the direct problem. Lots of methods have been studied during the last decades. Standard techniques include method of moment (MoM) [9] or the finite element-boundary integral (FE-BI) method [10]. High frequency asymptotic approaches include Gaussian beam shooting [6], the bounding and shooting ray method [12]. Recently, we also proposed a fast mode matching method for 2-D and 3-D large cavities [3,5]. Mathematical analysis for the cavity scattering problem can be seen in [1,2]. In order to satisfy both the efficiency and the accuracy, we adopt the method proposed in [4], which is based on finite difference with second order accuracy in the interior.

In the following section, the optimal design of a cavity with multilayered RAM is formulated as a minimization problem, with the constraint being formulated as a Helmholtz equation with artificial boundary condition. The objective function and design variables are also defined in this section. The continuous adjoint method is applied in Section 3 for the gradient of the backscatter RCS. Section 4 introduces the SQP optimization technique to the RCS reduction. Numerical experiments are provided in Section 5 to show the application of the method.

2 Problem formulation

Consider a rectangular cavity $\Omega=[0,a]\times[-b,0]$ embedded in a ground plane illuminated by a plane wave, as illustrated in Fig. 1. The problem is in 2-D by assuming the cavity and the materials are invariant in the z direction. Above the ground plane is empty space with dielectric permittivity ε_0. The surface of the ground plane Γ^c and the boundary S of the cavity are assumed to be perfect conductors. The cavity is filled with inhomogeneous material in layered structure with permittivities ε_i, $i=1,2,\cdots,n$, where n is the number