Commun. Comput. Phys.
doi: 10.4208/cicp.150313.051213s

Vol. **15**, No. 4, pp. 1167-1183
April 2014

A Nonlinear PIC Algorithm for High Frequency Waves in Magnetized Plasmas Based on Gyrocenter Gauge Kinetic Theory

Jian Liu[1,2], Zhi Yu[3,2,*] and Hong Qin[1,4]

[1] *Department of Modern Physics and Collaborative Innovation Center for Advanced Fusion Energy and Plasma Sciences, University of Science and Technology of China, Hefei, Anhui 230026, China.*
[2] *Key Laboratory of Geospace Environment, University of Science and Technology of China, Chinese Academy of Sciences, Hefei, Anhui 230026, China.*
[3] *Theory and Simulation Division, Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, Anhui 230031, China.*
[4] *Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543, USA.*

Received 15 March 2013; Accepted (in revised version) 5 December 2013

Communicated by Xueqiao Xu

Available online 21 January 2014

Abstract. Numerical methods based on gyrocenter gauge kinetic theory are suitable for first principle simulations of high frequency waves in magnetized plasmas. The δf gyrocenter gauge PIC simulation for linear rf wave has been previously realized. In this paper we further develop a full-f nonlinear PIC algorithm appropriate for the nonlinear physics of high frequency waves in magnetized plasmas. Numerical cases of linear rf waves are calculated as a benchmark for the nonlinear GyroGauge code, meanwhile nonlinear rf-wave phenomena are studied. The technique and advantage of the reduction of the numerical noise in this full-f gyrocenter gauge PIC algorithm are also discussed.

AMS subject classifications: 65Z05, 70K70, 70K75, 78M34

Key words: Gyrocenter gauge kinetics, multi-scale problem, nonlinear system, particle-in-cell.

---

1 Introduction

Radio frequency (rf) waves have been theoretically proposed and experimentally proved an effective method for plasma heating and current drive in magnetic confinement fu-

*Corresponding author. *Email addresses:* `jliuphy@ustc.edu.cn` (J. Liu), `yuzhi@ipp.ac.cn` (Z. Yu), `hongqin@ustc.edu.cn` (H. Qin)

sion (MFC) research [1–4]. Recent research shows strong evidences that toroidal plasma rotations can be induced by rf waves launching in tokamaks [5–7]. These facts reflect that rf waves launching is important to improve confinement and to maintain H-mode runs of tokamaks. On the other hand, our understanding on the physics of rf waves in magnetized plasmas is still limited. The multi-scale and nonlinear properties of rf physics in magnetized plasmas bring challenges to theoretical analysis. First principle simulations build a bridge between theories and experiments for rf wave research. To numerically study rf waves, an efficient algorithm is thus required. A δf particle-in-cell (PIC) algorithm based on gyrocenter gauge kinetic theory has been successfully applied to linear rf wave simulation [8]. In this paper, we further develop a full-f gyrocenter gauge PIC algorithm which is appropriate for the description of nonlinear rf wave physics.

Because the frequencies of rf waves are high enough, the high-frequency responses of charged particles in magnetized plasmas, as well as the changes of their gyro-orbits, have to be taken into account. This indicates that the length of time step in rf wave simulations should be extremely small compared with the time scale of the problem, which brings along heavy computational burdens. Traditional gyrokinetic theory improves numerical efficiency through averaging out the fast gyromotion of charged particles and only sustain the slow gyrocenter dynamics. Though traditional gyrokinetics is a powerful tool for low frequency physics, it cannot be applied to rf-wave simulation directly because the fast responses of charged particles are erased by gyro-average.

Gyrocenter gauge kinetic theory is a special version of the generalized kinetic theory, which deal with the Vlasov-Maxwell system in a geometric view [9–11]. Gyrocenter gauge kinetic theory is aimed to solve the multi-time-scale problems, such as the high frequency waves in magnetized plasmas. In the theory, fast gyromotion of charged particles is decoupled from slow gyrocenter dynamics instead of being removed by gyro-average. Then particle dynamics with different time-scales can be advanced separately in different time steps. When decoupling dynamics with different time-scales, the key step is to find a proper symmetry, which is the gyro-symmetry in the case of charged particles in magnetized plasmas. However, the existence of high frequency electromagnetic perturbations breaks the original gyro-symmetry. gyrocenter gauge kinetic theory resolves it through a second gyrocenter coordinate transform using Lie perturbation method.

Traditional gyrokinetics consists of two pivotal steps, gyrocenter coordinate transform and gyrophase average. Gyrocenter gauge kinetic theory shares the first step with traditional gyrokinetics. In this first gyrocenter coordinate transform, particle coordinate $(\mathbf{x},\mathbf{v})$ is transformed to unperturbed gyrocenter coordinate $\tilde{\mathbf{Z}}=(\tilde{\mathbf{X}},\tilde{u},\tilde{\mu},\tilde{\theta})$, where $\mathbf{x}$ is particle position coordinate, $\mathbf{v}$ is particle velocity coordinate, $\tilde{\mathbf{X}}$ is gyrocenter position coordinate, $\tilde{u}$ is parallel velocity, $\tilde{\mu}$ is magnetic moment, and $\tilde{\theta}$ is gyrophase. Yet in gyrocenter gauge kinetic theory, a second gyrocenter coordinate transform is needed to transform the unperturbed gyrocenter coordinate $\tilde{\mathbf{Z}}$ to the perturbed gyrocenter coordinate $\mathbf{Z}=(\mathbf{X},u,\mu,\theta)$. The second gyrocenter coordinate transform employs Lie coordinate perturbation method and thus requires assumption that high frequency perturbations are not strong enough to violently break the original gyro-symmetry. This assumption