

A TV-Based Iterative Regularization Method for the Solutions of Thermal Convection Problems

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Abstract. Linear/nonlinear and Stokes based-stabilizations for the filter equations for damping out primitive variable (PV) solutions corrupted by uniformly distributed random noises are numerically studied through the natural convection (NC) as well as the mixed convection (MC) environment. The most recognizable filter-scheme is based on a combination of the negative Laplace equation multiplied with the selection of the spatial scale and a linear function in order to preserve the uniqueness of the filtered solution. A more complicated filter-scheme, based on a Stokes problem which couples a filtered velocity and a filtered (artificial) pressure (or Lagrange multiplier) in order to enforce the incompressibility constraint, is also studied. Linear and Stokes based-filters via nested iterative (NI) filters and the consistent splitting scheme (CSS) are proposed for the NC/MC problems. Inspired by the total-variation (TV) model of image diffusion, well preserved feature flow patterns from the corrupted NC/MC environment are obtained by TV-Stokes based-filters together with the CSS. Our experimental results show that our proposed algorithms are effective and efficient in eliminating the unwanted spurious oscillations and preserving the accuracy of thermal convective fluid flows.

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Key words: Linear/nonlinear differential filter, Stokes differential filters, Leray-alpha model, total-variation regularization.

1 Introduction

The differential filter is one of the most straightforward regularization or stabilization methods in Computational Fluid Dynamics (see [7] for a review), where resolved amplitudes of fluctuations of fluid flows can be diminished. Let ζ and $\bar{\zeta}$ be an unfiltered and

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a filtered scalar or vector field variable, respectively. The model equation is based on a partial differential equation of evolution type described as follows (e.g., [8,9]):

Given ξ , find a unique solution $\bar{\xi}$ such that

$$\begin{cases} -\alpha^2 \nabla^2 \bar{\xi} + \bar{\xi} = \xi, & \text{in } \Omega, \\ \bar{\xi} = \xi, & \text{on } \partial\Omega, \end{cases} \quad (1.1)$$

where α , $0 < \alpha$, represents a regularization length (or spatial) scale. By adjusting α , the reduction of the unwanted spurious oscillations corrupted by uniformly distributed random noises, can be accomplished simply by the smoothing filter $(-\alpha^2 \nabla^2 + I)^{-1}$, where ∇^2 and I are the Laplace and identity operators, respectively.

When considering the incompressible flow problems, the extension to Eq. (1.1) is done by adding a filtered pressure gradient term as a Lagrange multiplier and enforcing the incompressibility constraint, often referred to as the Stokes differential filter (e.g., [10,12]). The method is described as follows:

Given ξ , find a pair of unique solutions $(\bar{\xi}, \lambda)$ such that

$$\begin{cases} -\alpha^2 \nabla^2 \bar{\xi} + \bar{\xi} + \nabla \lambda = \xi, & \text{in } \Omega, \\ \nabla \cdot \bar{\xi} = 0, & \text{in } \Omega, \\ \bar{\xi} = \xi, & \text{on } \partial\Omega, \end{cases} \quad (1.2)$$

where the filtered pressure λ is assumed known up to an arbitrary constant. It is known that the choice of the boundary conditions in (1.1) and (1.2) is an open problem. However, those that we used are the most reasonable choices and are widely accepted in literature.

Let us consider the NC as well as the MC environment. Assume at each time, the primitive variable (PV) $\xi = \{u, v, p, T\}$, where u and v are velocities, p is the pressure and T is the temperature, in this study is generated from the error-free one and is assumed to have random noise, as shown in Eq. (1.3)

$$\xi = \xi_{\text{free}} + \omega \hat{\sigma}, \quad (1.3)$$

where the term $\hat{\sigma}$ is the standard deviation of the random errors, which is supposed constant and ω is a random variable with normal distribution, zero mean, and unitary standard deviation, so that Eq. (1.3) describes an unpredictable error in the solution of ξ . In order to recover/preserve the fluid flow details as much as possible, the noise is removed in different successive filtering steps that consist of the following four-step strategy [13]:

At each time, evolve-filter-deconvolve-relax.

The four-step strategy can be sequentially described as follows:

1. The evolution step is used to determine each PV (or unfiltered) solution at the $n+1^{\text{th}}$ time-step from the n^{th} time-step.