Commun. Comput. Phys.
doi: 10.4208/cicp.190112.250512a

Vol. **13**, No. 5, pp. 1432-1454
May 2013

Gas Kinetic Scheme for Anisotropic Savage-Hutter Model

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Received 19 January 2012; Accepted (in revised version) 25 May 2012

Communicated by Kun Xu

Available online 8 October 2012

Abstract. The gas-kinetic scheme is applied to a depth-integrated continuum model for avalanche flows, namely the Savage-Hutter model. In this method, the continuum fluxes are calculated based on the pseudo particle motions which are relaxed from non-equilibrium to equilibrium states. The processes are described by the Bhatnagar-Gross-Krook (BGK) equation. The benefit of this scheme is its capability to resolve shock discontinuities sharply and to handle the vacuum state without special treatments. Because the Savage-Hutter equation bears an anisotropic stress on the tangential space of the topography, the equilibrium distribution function of the microscopic particles are shown to be bi-Maxwellian. These anisotropic stresses are the key to preserve the coordinate objectivity in the Savage-Hutter model. The effect of the anisotropic stress is illustrated by two examples: an axisymmetric dam break and a finite mass sliding on an inclined plane chute. It is found that the propagation of the flow fronts significantly depends on the orientation of the principal axes of the tangential stresses.

AMS subject classifications: 76A05, 82B40, 86-08, 97M10

Key words: Gas-kinetic scheme, BGK equation, extended Savage-Hutter equation.

1 Introduction

Landslides, avalanches, and debris flows are gravity-driven rapid geophysical flows. Because these flows commonly exhibit the characteristics of shallowness, the shallow-water

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type of equations are often applied to model the phenomena. In addition, the flows contain a large portion of solid particles and present solid-like behavior. To account for the solid effects, Savage & Hutter [1] propose to use the Mohr-Coulomb soil constitutive law for the landslide or avalanche materials in the shallow-water continuum model. When the granular mass is in motion, the basal friction causes the material to move on the basal surface to yield. With the internal friction angle and the Mohr-Coulomb yield criterion, the stresses in the tangential plane to the basal surface are calculated and incorporated into the momentum equation.

The Savage-Hutter model is subsequently extended for two-dimensional channel topography, Hutter *et al.* [2], Pudasaini & Hutter [3], Wang *et al.* [4]. The model equations form a conservative hyperbolic system. Because this type of partial differential equations contains discontinuous weak solutions, e.g. shock waves (hydraulic jumps), numerical schemes need to be able to resolve these weak solutions accurately. There are several common categories of solvers, for example, the high-order (approximate) Riemann solvers, LeVeque [5], Toro [6], and the non-oscillatory central finite difference schemes, Jiang *et al.* [7], Tai *et al.* [8]. In addition, particularly for landslide flow applications, wet-dry states are often encountered such as in the landslide flows of a finite mass reported in the previous literature.

In this paper, we aim to extend the kinetic scheme, Xu [9], to the Savage-Hutter model. In this method, the flow fluxes across the interfaces between computational cells are simulated by the motions of microscopic pseudo particles. These pseudo particles move along with their microscopic velocities and are subjected to perfectly elastic collisions. Under such circumstances, the density distribution function of the particles is assumed to follow the approximate Boltzmann equation, the Bhatnagar-Gross-Krook equation. Taking statistical moments of these pseudo particle motions yields the mass and momentum fluxes in the continuum regime. The benefit of this scheme is its capability to resolve shock discontinuities sharply, the positiveness of the flow depth, and to handle the wet-dry state without the need of special treatments.

With the Mohr-Coulomb soil constitutive law and through the theoretical derivation of the depth averaged model [2, 10–12], the stresses in the tangential plane to the basal surface are related to the flow depth and basal friction. The principal axes of the stresses are in general dependent on the local flow conditions, i.e. the tangential stresses are anisotropic. To simplify the complexity of this solid property for numerical simulations, Hutter *et al.* [2], Tai *et al.* [8], as well as Wang *et al.* [4] align the primary principal axis along the flow channel direction and the minor axes in the transverse direction. Though the approach simplifies the numerical scheme, the model becomes coordinate-dependent, as commented in Hutter, Wang and Pudasaini [13]. To amend this deficit, a variety of models have been proposed.

There are three main categories of these amending theories: Iverson and Denlinger [14] model the tangential stresses of a fluid element as isotropic active (passive) soil stresses if the element is in dilation (compaction). De Toni and Scotton [15], and Kelfoun and Druitt [16] assume that the primary principal axis of the tangential stresses is parallel