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A Coupled Approach for Fluid Dynamic Problems Using the PDE Framework Peano

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Abstract. We couple different flow models, i.e. a finite element solver for the Navier-Stokes equations and a Lattice Boltzmann automaton, using the framework Peano as a common base. The new coupling strategy between the meso- and macroscopic solver is presented and validated in a 2D channel flow scenario. The results are in good agreement with theory and results obtained in similar works by Latt et al. In addition, the test scenarios show an improved stability of the coupled method compared to pure Lattice Boltzmann simulations.

AMS subject classifications: 35Q30, 35Q35, 76M10, 76-xx

Key words: Spatial coupling, Lattice Boltzmann, Navier-Stokes, Peano.

1 Introduction

Many flow systems – especially in micro- and nano-fluidics – are strongly influenced by physical processes that appear on different spatial and temporal scales. Flows through nanopores influenced by Brownian motion or flows in porous media are typical examples. Solving these kinds of complex systems by means of computational fluid dynamics (CFD) often requires highly adaptive concepts or different sorts of solvers depending on the current scale to be simulated.

With respect to Navier-Stokes and Lattice Boltzmann related solver techniques, detailed discussions and comparisons of their performance and qualitative behaviour can amongst others be found in [8, 13] and should only be touched here. Considering flow problems on mesoscopic scales, Lattice Boltzmann methods have been validated for a wide range of scenarios such as particulate flows [10] or fluctuating hydrodynamics

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[7]. Furthermore, as discussed in our previous works [13], simulation tests by different groups in the finite Knudsen number regime revealed strong deviations of macroscopic Navier-Stokes and Burnett descriptions from the physical phenomena (cf. [17,18]) whereas mesoscopic approaches like Direct Simulation Monte Carlo and Lattice Boltzmann methods still yielded feasible results [9, 21]. On the other hand, Navier-Stokes systems might e.g. be superior in case of (laminar) convection dominated flows where classical Lattice Boltzmann automata often require even finer grid resolutions and small timesteps. Implicit solver strategies for the Navier-Stokes equations would allow for considerably bigger timesteps in these cases.

With implementations of a Navier-Stokes and a Lattice Boltzmann solver integrated within the Peano framework, we already deeply discussed and evaluated the single codes in [13]; it is in this context that the idea emerged to spatially couple both approaches and exploit the advantages of both schemes as also proposed by Latt et al. in [12]. There are different scenarios, where this idea might become favorable. Amongst others, considering fluid-structure interaction problems in the laminar flow regime, the weakly compressible Lattice Boltzmann method could be used to stabilize pressure induced perturbations in the surrounding of the respective structure, whereas the Navier-Stokes equations are solved further away from the structure. Besides, in the case of simulations of multiscale flow problems, like laminar flows near and through porous structures such as membranes, a spatial coupling of Navier-Stokes and Lattice Boltzmann solvers might allow to resolve the complex geometry of the porous medium and describe this part of the flow system by means of Lattice Boltzmann and – on a coarser grid – to compute the rest of the flow domain efficiently by (implicit) Navier-Stokes solver techniques.

There have already been several works on this field (see for example [1,12]). It is particularly the work presented in [12] where the coupling between a finite difference Navier-Stokes solver and a Lattice Boltzmann automaton is established and validated. Though parts of the respective algorithmic realisations carry over to our coupling approach, we want to focus on a new method for the macro-to-mesoscale coupling which is based on a minimisation procedure. We particularly point out the feasibility of coupling different types of solvers within the Peano framework.

This paper is structured as follows: The theoretical foundation for the Lattice Boltzmann method and our Navier-Stokes solver is briefly reviewed in Section 2. Besides, the Chapman-Enskog expansion connecting the meso- and macroscopic approach is carried out in order to obtain the conservation laws that need to be fulfilled at the interface between the two solvers. In Section 3, we give a short introduction to the framework Peano and discuss its basic properties at the example of our Lattice Boltzmann implementation. In Section 4, we discuss the underlying grid topology and afterwards focus on the methodology for coupling Navier-Stokes to Lattice Boltzmann and vice versa. A description of the underlying implementations within the Peano framework is given in Section 5. Results for the coupling applied to pure channel flow are provided in Section 6. Finally, we close the discussion and give a short conclusion and outlook on future work in Section 7.