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Modeling Ionic Polymer-Metal Composites with Space-Time Adaptive Multimesh hp-FEM

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Abstract. We are concerned with a model of ionic polymer-metal composite (IPMC) materials that consists of a coupled system of the Poisson and Nernst-Planck equations, discretized by means of the finite element method (FEM). We show that due to the transient character of the problem it is efficient to use adaptive algorithms that are capable of changing the mesh dynamically in time. We also show that due to large qualitative and quantitative differences between the two solution components, it is efficient to approximate them on different meshes using a novel adaptive multimesh hp-FEM. The study is accompanied with numerous computations and comparisons of the adaptive multimesh hp-FEM with several other adaptive FEM algorithms.

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1 Introduction

Ionic Polymer-Metal Composites (IPMC) have been studied during the past two decades for their potential to serve as noiseless mechanoelectrical and electromechanical transducers [1, 4, 5, 7, 9, 10, 17]. The advantages of IPMC over other electroactive polymer

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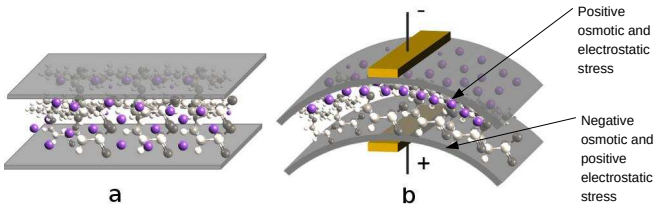

Figure 1: Conceptual model of the actuation of IPMC. Initial counter ion distribution (a) and the distribution and resulting bending after applying a voltage (b).

actuators are low voltage bending, high strains ($>1\%$), and an ability to work in wet environments. A typical IPMC consists of a thin sheet of polymer (often Nafion or Teflon) which is sandwiched between noble metal electrodes such as platinum or gold. When fabricated, the polymer membrane is saturated with certain solvent and ions such as water and H^+. When a voltage is applied to the electrodes, the counter ions start migrating due to the imposed electric field. By dragging along the solvent, the osmotic pressure difference near the electrodes results in bending of the material (see Fig. 1).

In this study we will model IPMC materials via a multiphysics coupled problem consisting of the Poisson and Nernst-Planck equations (abbreviated by PNP in the following). These equations are used to model charge transport in materials that include ionic migration, diffusion, and convection. The charge transport process is a key mechanism for electromechanical transduction.

The PNP system is highly nonlinear and for a typical domain with two electrodes, largest differences in charge concentration occur in a very narrow region near the boundary. The computing power required for a full scale problem is significant [8]. This is why we are interested in exploring adaptive algorithms-we hope to obtain meshes that are optimal in terms of calculation time and calculation error.

The Nernst-Planck equation for a mobile species- in our case for counter ions-has the form

$$\frac{\partial C}{\partial t}+\nabla\cdot(-D\nabla C-\mu FC\nabla\phi)=0. \tag{1.1}$$

Here C stands for the counter ion concentration with the initial value of C_0, D is diffusion, μ mobility, F Faraday constant and ϕ voltage. We have neglected the velocity of the species as in our case it can be assumed zero. The Poisson equation has the form

$$-\nabla^2\phi=\frac{F\rho}{\varepsilon}, \tag{1.2}$$

where ε is the absolute dielectric permittivity. The charge density $\rho=C-C_0$ where C_0 is a constant anion concentration.

The outline of the paper is as follows: Section 2 shows that the solution components C and ϕ have very different behavior, which is the reason why it is difficult to find a common mesh that would be optimal for both of them. This explains why we are interested in approximation them on individual meshes equipped with mutually independent