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Points of Coincidence and Common Fixed Points for II-Expansive Mappings on Complex Valued Metric Spaces

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Abstract. We use the two mappings satisfying II-expansive conditions on complex valued metric spaces to construct the convergent sequences and prove that the unique limit of the sequences is the point of coincidence or common fixed point of the two mappings. Also, we discuss the uniqueness of points of coincidence or common fixed points and give the existence theorems of unique fixed points. The obtained results generalize and improve the corresponding conclusions in references.

Key Words: Complex valued metric space, II-expansive mapping, Cauchy sequence, point of coincidence, common fixed point.

AMS Subject Classifications: 47H05, 47H10, 54E40, 54H25

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1 Introduction and preliminaries

Real metric spaces have been widely generalized and improved by cone metric spaces [1] and topological vector space-valued cone metric spaces [2, 3] and so on. A number of authors discussed and obtained some fixed point and common fixed point theorems on these spaces, greatly generalized and improved some corresponding results.

Recently, the author in [4] defined complex valued metric spaces on a nonempty set X and obtained coincidence point theorems and common fixed point theorems for two mappings satisfying some contractive conditions on this space. The authors in [5–7] generalized and extended the results in [4]. The authors [8, 9] discussed the existence problems of common fixed points for two mappings satisfying expansive conditions. These results further enrich and improve the existence theory of coincidence points and common fixed points on complex valued metric spaces.

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In this paper, by weakening the condition and using a new method of proof, we generalize a known result [9, Theorem 3.1] and give another unique common fixed point theorem for two mappings satisfying a II-expansive condition.

In what follows, we recall some notations and definitions that will be utilized in our subsequent discussion.

Let $\mathbb{C}$ be the set of complex numbers and $z_1,z_2\in\mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1\precsim z_2 \Leftrightarrow [\mathrm{Re}(z_1)\le \mathrm{Re}(z_2)]\wedge[\mathrm{Im}(z_1)\le \mathrm{Im}(z_2)].$$

Consequently, $z_1\precsim z_2$ if and only if one of the following conditions is satisfied:

(C1) $\mathrm{Re}(z_1)=\mathrm{Re}z_2$, $\mathrm{Im}z_1=\mathrm{Im}z_2$;

(C2) $\mathrm{Re}(z_1)<\mathrm{Re}z_2$, $\mathrm{Im}z_1=\mathrm{Im}z_2$;

(C3) $\mathrm{Re}(z_1)=\mathrm{Re}z_2$, $\mathrm{Im}z_1<\mathrm{Im}z_2$;

(C4) $\mathrm{Re}(z_1)<\mathrm{Re}z_2$, $\mathrm{Im}z_1<\mathrm{Im}z_2$.

In particular, we write $z_1\precnsim z_2$ if $z_1\neq z_2$ and one of (C2), (C3), (C4) is satisfied, and we write $z_1<z_2$ if only (C4) is satisfied.

Obviously, the following statements hold:

(i) If $b\ge a\ge 0$, then $az\precsim bz$ for any $z\in\mathbb{C}$ with $0\precsim z$;

(ii) if $0\precsim z_1\precnsim z_2$, then $|z_1|<|z_2|$;

(iii) if $z_1\precsim z_2$ and $z_2<z_3$, then $z_1<z_3$;

(iv) if $z_1\precsim z_2$ and $z\in\mathbb{C}$, then $z+z_1\precsim z+z_2$.

Definition 1.1 (see [4–7])**.** Let X be a nonempty set. If a mapping $d:X\times X\to\mathbb{C}$ satisfies the following conditions:

(i) $0\precsim d(x,y)$ for all $x,y\in X$, and $d(x,y)=0$ if and only if $x=y$;

(ii) $d(x,y)=d(y,x)$ for all $x,y\in X$;

(iii) $d(x,z)\precsim d(x,y)+d(y,z)$ for all $x,y,z,\in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Example 1.1 (see [4])**.** Let $X=\mathbb{C}$. Define a mapping $d:X\times X\to\mathbb{C}$ as follows

$$d(z_1,z_2)=e^{ik}|z_1-z_2|,\quad \forall z_1,z_2\in X,$$

where $k\in\mathbb{R}$. Then (X,d) is a complex valued metric space.

Example 1.2. Let $X=\{a,b,c\}$. Define a mapping $d:X\times X\to\mathbb{C}$ by

$$\begin{aligned} &d(a,a)=d(b,b)=d(c,c)=0, && d(a,b)=d(b,a)=3+4i,\\ &d(a,c)=d(c,a)=2+3i, && d(b,c)=d(c,b)=4+5i. \end{aligned}$$

Obviously, (X,d) is a complex valued metric space.