Analysis in Theory and Applications
Anal. Theory Appl., Vol. **33**, No. 3 (2017), pp. 287-300

DOI: 10.4208/ata.2017.v33.n3.8

Direct and Reverse Carleson Conditions on Generalized Weighted Bergman-Orlicz Spaces

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Received 13 April 2017; Accepted (in revised version) 14 July 2017

Abstract. Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$. For $\alpha>-1$, let $dA_\alpha(z)=(1+\alpha)\left(1-|z|^2\right)^\alpha dA(z)$ be the weighted Lebesgue measure on $\mathbb{D}$. For a positive function $\omega\in L^1(\mathbb{D},dA_\alpha)$, the generalized weighted Bergman-Orlicz space $\mathcal{A}^\psi_\omega(\mathbb{D},dA_\alpha)$ is the space of all analytic functions such that

$$\|f\|^\psi_\omega=\int_{\mathbb{D}}\psi(|f(z)|)\omega(z)dA_\alpha(z)<\infty,$$

where ψ is a strictly convex Orlicz function that satisfies other technical hypotheses. Let G be a measurable subset of $\mathbb{D}$, we say G satisfies the reverse Carleson condition for $\mathcal{A}^\psi_\omega(\mathbb{D},dA_\alpha)$ if there exists a positive constant C such that

$$\int_G\psi(|f(z)|)\omega(z)dA_\alpha(z)\ge C\int_{\mathbb{D}}\psi(|f(z)|)\omega(z)dA_\alpha(z),$$

for all $f\in\mathcal{A}^\psi_\omega(\mathbb{D},dA_\alpha)$. Let μ be a positive Borel measure, we say μ satisfies the direct Carleson condition if there exists a positive constant M such that for all $f\in\mathcal{A}^\psi_\omega(\mathbb{D},dA_\alpha)$,

$$\int_{\mathbb{D}}\psi(|f(z)|)d\mu(z)\le M\int_{\mathbb{D}}\psi(|f(z)|)\omega(z)dA_\alpha(z).$$

In this paper, we study the direct and reverse Carleson condition on the generalized weighted Bergman-Orlicz space $\mathcal{A}^\psi_\omega(\mathbb{D},dA_\alpha)$. We present conditions on the set G such that the reverse Carleson condition holds. Moreover, we give a sufficient condition for the finite positive Borel measure μ to satisfy the direct carleson condition on the generalized weighted Bergman-Orlicz spaces.

Key Words: Orlicz function, global Δ_2-condition, reverse Carleson condition, Direct Carleson condition, closed range, Pseudohyperbolic disks, Orlicz spaces, weighted Bergman spaces, generalized weighted Bergman-Orlicz spaces.

AMS Subject Classifications: 46E15, 30C25, 30H05, 46E38, 30C80, 32C15

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1 Introduction

Let $\mathbb{D}$ be the open unit disk $\{z\in\mathbb{D}:|z|<1\}$ in the complex plane $\mathbb{C}$. Let $dA(z)=\frac{1}{\pi}dxdy$ be the normalized Lebesgue area measure on $\mathbb{D}$. For $\alpha>-1$, let $dA_\alpha(z)=(1+\alpha)\left(1-|z|^2\right)^\alpha dA(z)$ be the weighted Lebesgue area measure on $\mathbb{D}$. As usual, we denote by $\mathcal{H}(\mathbb{D})$ the space of all analytic functions on $\mathbb{D}$. An Orlicz function is a real-valued, continuous, increasing function $\psi:[0,\infty)\to[0,\infty)$ with $\psi(0)=0$ and $\lim_{t\to\infty}\psi(t)=\infty$. For a finite positive measure μ on $\mathbb{D}$, the Orlicz space $L^\psi(\mathbb{D},d\mu)$ is the space of all measurable functions f such that

$$\int_{\mathbb{D}}\psi(\lambda|f(z)|)d\mu(z)<\infty,$$

for some positive constant λ depending on f. It is well known that $L^\psi(\mathbb{D},d\mu)$ is a Banach space under the following (quasi-)norm

$$\|f\|_\psi^{lux}=\inf\left\{C>0:\int_{\mathbb{D}}\psi\left(\frac{|f(z)|}{C}\right)d\mu(z)\le 1\right\}.$$

This (quasi-)norm is known as Luxemburg norm.

We say a function ψ satisfies the global Δ_2-condition if for every $r\ge 0$ there is a constant $K>1$ such that $\psi(rt)\le K\psi(t)$ for all $t\ge 0$. It is well know that if ψ is convex, then $L^\psi(\mathbb{D},d\mu)$ becomes the space of all functions f such that

$$\int_{\mathbb{D}}\psi(|f(z)|)d\mu(z)<\infty.$$

For more information about Orlicz spaces, we refer the reader to the monograph [28], the memoirs [17] and the references therein.

In this paper, we assume $\psi:[0,\infty)\to[0,\infty)$ is strictly convex Orlicz function satisfying the global Δ_2-condition. Moreover, we will assume

$$\lim_{t\to\infty}\frac{\psi(t)}{t}=\infty,$$

this condition is essential to exclude the case $\psi(t)=at$ for some $a>0$.

For a positive function $\omega\in L^1(\mathbb{D},dA_\alpha)$, the generalized weighted Bergman-Orlicz space $\mathcal{A}_\omega^\psi(\mathbb{D},dA_\alpha)$ is the space of all functions $f\in\mathcal{H}(\mathbb{D})$ such that

$$\|f\|_\omega^\psi=\int_{\mathbb{D}}\psi(|f(z)|)\omega(z)dA_\alpha(z)<\infty.$$

Hence, the weighted Bergman-Orlicz space is defined as the set

$$\mathcal{A}_\omega^\psi(\mathbb{D},dA_\alpha)=\mathcal{H}(\mathbb{D})\cap L^\psi(\mathbb{D},\omega dA_\alpha).$$