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On Approximation Properties of Modified Sázas-Mirakyan Operators via Jain Operators

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Abstract. In the present manuscript, we propose the modification of Jain operators which the generalization of Szász-Mirakyan operators. These new class operators are linear positive operators of discrete type depending on a real parameters. We give theorem of degree of approximation and the Voronovskaya asymptotic formula.

Key Words: Positive linear operators, Jain operators, Szász-Mirakyan operator.

AMS Subject Classifications: 41A25, 41A30, 41A36

1 Introduction

In [1], Patel and Mishra introduced following sequence of positive linear operators, for $f\in C([0,\infty))$; $0\le\mu<1$; $1<\gamma\le e$

$$P_n^{[\mu,\,\gamma]}(f,x)=\sum_{k=0}^{\infty}\omega_{\mu,\gamma}(k,nx)f\left(\frac{k}{n}\right), \tag{1.1}$$

where

$$\omega_{(\mu,\gamma)}(k,nx)=nx(\log\gamma)^k(nx+k\mu)^{k-1}\frac{\gamma^{-(nx+k\mu)}}{k!}.$$

In the particular case, $\gamma=e$ the operators (1.1) equal to Jain operators [2]. Also, for $\gamma=e$ and $\mu=0$, the operators (1.1) turns to classical the Szász-Mirakyan operators. Therefore,

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the above operators is the generalization of Szaász-Mirakyan operators via Jain operators. The relation between the local smoothness of function and local approximation, the degree of approximation and the statistical convergence of the Jain operators was studied by Agratini [3]. Umar and Razi [4] studied Kantorovich-type extension of Jain operators. Durrmeyer type generalization of Jain operators and its approximation properties was elaborated by Tarabie [5], Mishra and Patel [6], Patel and Mishra [7] and Agratini [8]. Some related work in this area can be found in [9–16]. Motivated by such operators, we further generalized following modification of the operators (1.1) as: For $f\in C([0,\infty))$; $n\in\mathbb{N}$; $1<\gamma\le e$; $0\le\mu<1$;

$$P_n(f,x):=P_n^{[\mu,a_n,b_n,\gamma]}(f,x)=\sum_{k=0}^{\infty}\omega_{(\mu,\gamma)}(k,a_nx)f\left(\frac{k}{b_n}\right), \tag{1.2}$$

where $\omega_{(\mu,\gamma)}(k,a_nx)$ as defined in (1.1) and $(a_n)_{n=1}^{\infty}$ and $(b_n)_{n=1}^{\infty}$ are given increasing and unbounded numerical sequence such that $a_n\ge 1$, $b_n\ge 1$ and $\left(\frac{a_n}{b_n}\right)$ is nondecreasing and

$$\frac{a_n}{b_n}=1+o\left(\frac{a_n}{b_n}\right). \tag{1.3}$$

Along the paper, when we will deal with approximation results, the parameters $\mu\in[0,1)$ and $\gamma\in(1,e]$ will be assumed to be a sequence μ_n and γ_n which tends to zero and Euler's number e as $n\to\infty$, respectively.

2 Moments of P_n

To discuss moments of the operators (1.2), we need following lemmas:

Lemma 2.1 (see [1]). *For $0<\alpha<\infty$, $0\le\mu<1$ and $1<\gamma\le e$. Let*

$$\omega_{(\mu,\gamma)}(k,\alpha)=\alpha(\log\gamma)^k(\alpha+k\mu)^{k-1}\frac{\gamma^{-(\alpha+k\mu)}}{k!}. \tag{2.1}$$

Then

$$\sum_{k=0}^{\infty}\omega_{(\mu,\gamma)}(k,\alpha)=1. \tag{2.2}$$

Lemma 2.2 (see [1]). *Let $0<\alpha<\infty$, $0\le\mu<1$ and $1<\gamma\le e$. Suppose that*

$$S(r,\alpha,\mu,\gamma)=\sum_{k=0}^{\infty}\frac{1}{k!}(\log\gamma)^k(\alpha+k\mu)^{k+r-1}\gamma^{-(\alpha+k\mu)} \tag{2.3}$$

and

$$S(1,\alpha,\mu,\gamma)=1. \tag{2.4}$$