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On Fatou Type Convergence of Convolution Type Double Singular Integral Operators

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Dedicated to my academic father Professor Paul L. Butzer

Abstract. In this paper, some approximation formulae for a class of convolution type double singular integral operators depending on three parameters of the type

$$(T_\lambda f)(x,y)=\int_a^b\int_a^b f(t,s)K_\lambda(t-x,s-y)dsdt,\quad x,y\in(a,b),\quad \lambda\in\Lambda\subset[0,\infty), \tag{0.1}$$

are given. Here f belongs to the function space $L_1(\langle a,b\rangle^2)$, where $\langle a,b\rangle$ is an arbitrary interval in $\mathbb{R}$. In this paper three theorems are proved, one for existence of the operator $(T_\lambda f)(x,y)$ and the others for its Fatou-type pointwise convergence to $f(x_0,y_0)$, as (x,y,λ) tends to (x_0,y_0,λ_0). In contrast to previous works, the kernel functions $K_\lambda(u,v)$ don't have to be 2π-periodic, positive, even and radial. Our results improve and extend some of the previous results of [1,6,8,10,11,13] in three dimensional frame and especially the very recent paper [15].

Key Words: Fatou-type convergence, convolution type double singular integral operators, μ-generalized Lebesgue point.

AMS Subject Classifications: 41A35, 44A35, 42A85

1 Introduction

It is well-known that convolution type singular integral operators of various types create an important subject of mathematical investigations and are often applicable in physics. In particular, linear singular integrals of the convolution type occur in approximation theory, as a solutions of partial differential equations and in the theory of Fourier series. Let us mention, for example, the Fejér, Abel-Poisson, Poisson and Gauss-Weierstrass singular integrals (see [5]). All of these convolution type linear singular integrals which we mentioned above are depend on two parameters.

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It was Taberski [6] who took the first step to obtain some approximation results for linear singular integral operators depending on two parameters, which contain the single parameter sequence of convolution type singular integral operators widely developed in [5].

Taberski has shown that the pointwise convergence at which the points are Lebesgue points of integrable functions in $L_1(\langle -\pi,\pi\rangle)$, by the family of convolution type singular integral operators of the form

$$U(f;x,\lambda)=\int_{-\pi}^{\pi} f(t)K(t-x,\lambda)dt, x\in(-\pi,\pi), \tag{1.1}$$

where $K(t,\lambda)$ is a kernel which satisfies suitable assumptions. His results are valid on some subsets of the plane, i.e., Fatou type convergence was discussed. Since that time a new theory has been developed in order to give a unitary approach to the study of convergence and of the order of approximation for a general family of integral operators. In various papers, based mainly on Taberski's idea, some extensions and generalizations of the pointwise convergence theorems for the operators (1.1) and for some kind of its nonlinear counterpart have been given (see for example [1–4, 8–10] and [13]).

In papers [11–14] and [15] pointwise convergence of integrable functions in $L_1([-a,a]\times[-b,b])$ and $L_1(\langle -\pi,\pi\rangle^2)$ by a three parameter family of convolution type singular integral operators of the form

$$U(f;x,\lambda)=\int_{-a}^{a}\int_{-b}^{b} f(t,s)K_\lambda(t-x,s-y)dsdt, \quad (x,y)\in([-a,a],[-b,b]), \tag{1.2}$$

where $\lambda\in\Lambda\subset[0,\infty)$, have been investigated.

In the present paper, we extend and generalized the very recent results of Yilmaz [15], in which the author considers the operators (1.2) with $f\in L_1(\langle -\pi,\pi\rangle^2)$ and with a special kind of kernel, namely with a radial kernel, and obtained the pointwise convergence of (1.2) to $f(x_0,y_0)$ at a continuity point (x_0,y_0) of f.

In particular, we obtain precise approximation formulae for the Fatou-type pointwise convergence of $(T_\lambda f)(x,y)$ to $f(x_0,y_0)$ in $L_1(\langle a,b\rangle^2)$, by a family of convolution type singular integral operators of the form

$$(T_\lambda f)(x,y)=\int_a^b\int_a^b f(t,s)K_\lambda(t-x,s-y)dsdt, \quad x,y\in(a,b), \quad \lambda\in\Lambda\subset[0,\infty).$$

We note that, in this paper three theorems are proved, one for existence of the operator $(T_\lambda f)(x,y)$ and the others for its Fatou-type pointwise convergence to $f(x_0,y_0)$, as (x,y,λ) tends to (x_0,y_0,λ_0). In contrast to previous works, the kernel functions $K_\lambda(u,v)$ don't have to be 2π-periodic, positive, even and radial. Our results improve and extend some of the previous results of [1, 6, 8, 10, 11, 13] in three dimensional frame and especially the very recent paper [15]. Moreover, taking $\lambda\in\mathbb{N}$, $x=x_0$, and $K\geq 0$ this two-parameters family is reducedto the single parameter sequence of convolution type singular integral operators widely developed in [5].