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Standing Waves of the Coupled Nonlinear Schrödinger Equations

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Abstract. In this paper, we study the existence of standing waves of the coupled nonlinear Schrödinger equations. The proofs of which rely on the Lyapunov-Schmidt methods and contraction mapping principle are due to F. Weinstein in [1].

Key Words: Coupled nonlinear Schrödinger equations, Lyapunov-Schmidt, contraction mapping principle.

AMS Subject Classifications: 35B40, 35B45, 35Q55

1 Introduction

Nonlinear Schrödinger equations (NLS) have been broadly investigated in many aspects, such as concentraction and multi-bump phenomena for semiclassical states, existence of solitary waves (see [10]).

Recently, there are also many results for coupled Nonlinear Schrödinger equations (CNLS). One can refer to [2–8], for example, [3] studied the interaction and configuration of spikes in a doubled condensate by analyzing least energy solutions of two coupled nonlinear Schrödinger equations. It is shown that the interaction term determines the locations of the two spikes and asympototic shape of least energy solutions. [8] deals with a class of nonlinear Schrödinger equations which are linearly coupled, and have attracted a considerable attention in the last years.

In [1], F. Weinstein used Lyapunov-Schmidt methods is to prove the existence of s-tanding wave of Schrödinger equation. This method is classic. In this paper, we try to use the Lyapunov-Schmidt method to prove the case of equations, i.e., we want to use

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Lyapunov-Schmidt method to prove the existence of the coupled nonlinear Schrödinger equations

$$\begin{cases} \dfrac{h^2}{2m}\varphi_{xx}-P(x)|\phi|^2\varphi+\lambda\varphi+\gamma|\varphi|^2\varphi=ih\varphi_t, \\ \dfrac{h^2}{2m}\phi_{xx}-Q(x)|\varphi|^2\phi+\lambda\phi+\gamma|\phi|^2\phi=ih\phi_t. \end{cases} \tag{1.1}$$

We shall find solutions of the form

$$(\varphi(x,t),\phi(x,t))=\Big(\exp\Big(-\frac{iEt}{h}\Big)v_1(x),\exp\Big(-\frac{iEt}{h}\Big)u_1(x)\Big),$$

where u_1, v_1 are real-valued, then we get

$$\begin{cases} \dfrac{h^2}{2m}v_1''-P(x)v_1u_1^2+\lambda v_1+v_1^3=0, \\ \dfrac{h^2}{2m}u_1''-Q(x)v_1^2u_1+\lambda u_1+u_1^3=0. \end{cases} \tag{1.2}$$

For simplicity of notation, we shall assume that $m=1$, $\gamma=1$, so that (1.2) is reduced to

$$\begin{cases} \dfrac{h^2}{2}v_1''-P(x)u_1^2v_1+\lambda v_1+v_1^3=0, \\ \dfrac{h^2}{2}u_1''-Q(x)u_1v_1^2+\lambda u_1+u_1^3=0. \end{cases} \tag{1.3}$$

We assume $P(x)$, $Q(x)$ satisfy the following conditions:

(1) $P(x)$, $Q(x)$ are bounded, continuous and nondegenerate functions;

(2) $P(0)=Q(0)$, $\lim_{x\to\infty}P(x)=\lim_{x\to\infty}Q(x)=0$;

(3) $\max_{x\in R}P(x)<\lambda$, $\max_{x\in R}Q(x)<\lambda$.

The main result of this paper is:

Theorem 1.1. *For each nondegenerate critical point x_0 of P, Q, where P, Q satisfy the above conditions, there is h_0 such that for all h with $0<h<h_0$, the Eqs.* (1.3) *have a nonzero solution; and these solutions become more and more concentrated about x_0 as $h\to 0$.*

Some notations are the following:

$$\begin{aligned} &H=H^2, \quad L=L^2, \quad \langle,\rangle : L^2-\text{inner product}, \\ &\|f\|^2=\int f^2(x)dx, \quad K_{z,h}=span\{u'_{z,h}\}, \quad E_{z,h}=span\{v'_{z,h}\}, \\ &K^{\perp}_{z,h}=L^2-\text{orthogonal complement of } K_{z,h} \text{ in } H, \\ &\pi^{\perp}_{z,h}=L^2-\text{orthogonal projection to } K^{\perp}_{z,h}\times E^{\perp}_{z,h} \text{ in } H\times H. \end{aligned}$$