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Weighted Lipschitz Estimate for Commutator of Bochner-Riesz Operators on Weighted Morrey Spaces

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Abstract. In this paper, we will use a method of sharp maximal function approach to show the boundedness of commutator $[b,T_R^\delta]$ by Bochner-Riesz operators and the function b on weighted Morrey spaces $L^{p,\kappa}(\omega)$ under appropriate conditions on the weight ω, where b belongs to Lipschitz space or weighted Lipschitz space.

Key Words: Bochner-Riesz operator, weighted Morrey space, Lipschitz function.

AMS Subject Classifications: 42B25

1 Introduction and definitions

The Bochner-Riesz operator T_R^δ in $\mathbb{R}^n$ is defined in terms of Fourier transform by

$$(T_R^\delta f)^\wedge(\xi)=\Big(1-\frac{|\xi|^2}{R^2}\Big)_+^\delta f^\wedge(\xi),\quad R>0,$$

where $\hat{f}$ denotes the Fourier transform of f. And the maximal Bochner-Riesz operator is defined by

$$(T_*^\delta)(x)=\sup_{R>0}|(T_R^\delta)(x)|.$$

It is well known that $T_R^\delta=(f*\phi_{1/R})(x)$ is a convolution operator with the kernel $\phi_{1/R}$ [1], where

$$\phi(x)=\pi^{-\delta}\Gamma(\delta+1)|x|^{-(\frac{n}{2})+\delta}J_{\frac{n}{2}+\delta}(2\pi|x|),\quad \phi_{1/R}=R^n\cdot\phi(Rx)$$

and $J_\mu(t)$ is the Bessel function,

$$J_\mu(t)=\frac{(\frac{t}{2})^\mu}{\Gamma(\mu+\frac{1}{2})\Gamma(\frac{1}{2})}\int_{-1}^1 e^{its}(1-s^2)^{\mu-\frac{1}{2}}ds.$$

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The inequality

$$|\phi(x)|+|\nabla\phi(x)|\leq\frac{C}{(1+|x|)^{\delta+\frac{n+1}{2}}} \tag{1.1}$$

holds for ϕ^{δ}.

Bochner-Riesz operators have been investigated by many authors. Lee [16], Tao [17] and many others studied the so-called Bochner-Riesz conjecture, i.e., if $p>1$ and

$$\delta>\delta(p):=\max\Big\{n\Big|\frac{1}{p}-\frac{1}{2}\Big|-\frac{1}{2},0\Big\},$$

then T_R^{δ} is bounded on L^p. On the other hand, there are also many results concerning the weighted inequalities for them, see [18,19].

Let b be a locally integrable function and T_R^{δ} the Bochner-Riesz operator T_R^{δ}, we define the commutator operator by Bochner-Riesz operator

$$[b,T_R^{\delta}]f(x)=b(x)T_R^{\delta}f(x)-T_R^{\delta}(bf)(x).$$

Wang [18] proved that and $T_R^{\delta}(\delta)(\delta>(n-1)/2)$ is a bounded operator on the weighted Morrey spaces $L^{p,\kappa}(\omega)$ for $1<p<\infty$ and $0<\kappa<1$. In 2013, we proved the boundedness of the commutator of Bochner-Riesz operators and weighted BMO functions.

In 2009, Komori and Shirai [2] defined Morrey space $L^{p,\kappa}(\omega)$ and investigated the boundedness of classical operators in harmonic analysis, that is, the Hardy-Littlewood maximal operators, Calderón-Zygmund operators,the fractional integral operators, etc.

First we shall define the weighted Morrey space.

Definition 1.1 (see [2])**.** Let $1\leq p<\infty$, $0<\kappa<1$ and ω be a weight. then the weighted Morrey space is defined by

$$L^{p,\kappa}(\omega)=\{f\in L^p_{loc}(\omega):\|f\|_{L^{p,\kappa}(\omega)}<\infty\},$$

where

$$\|f\|_{L^{p,\kappa}(\omega)}=\sup_Q\Big(\frac{1}{\omega(Q)^{\kappa}}\int_Q|f(x)|^p\omega(x)dx\Big)^{\frac{1}{p}},$$

and the supermum is taken over all balls Q in $\mathbb{R}^n$.

Let $1\leq p<\infty$, $0<\kappa<1$. For two weights u and v, a weighted Morrey space with two weights is defined by

$$L^{p,\kappa}(u,v)=\{f\in L^p_{loc}(u):\|f\|_{L^{p,\kappa}(u,v)}<\infty\},$$

where

$$\|f\|_{L^{p,\kappa}(u,v)}=\sup_Q\Big(\frac{1}{v(Q)^{\kappa}}\int_Q|f(x)|^p u(x)dx\Big)^{\frac{1}{p}},$$

and the supermum is taken over all balls Q in $\mathbb{R}^n$.