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Coefficient Inequalities for p-Valent Functions

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Abstract. In the present paper, the authors introduce a new subclass of p-valent analytic functions with complex order defined on the open unit disk $\mathbb{U}=\{z:z\in\mathbb{C}$ and $|z|<1\}$ and obtain coefficient inequalities for the functions in these class. Application of these results for the functions defined by the convolution are also obtained.

Key Words: p-valent function, subordination, coefficient inequalities, convolution.

AMS Subject Classifications: 30C45

1 Introduction and definition

Let $\mathcal{A}_p(p\in\mathbb{N}:=\{1,2,3,\cdots\})$ be the class of functions $f(z)$ of the form

$$f(z)=z^p+\sum_{n=p+1}^{\infty}a_nz^n \tag{1.1}$$

that are regular and p-valent in the open unit disk

$$\mathbb{U}=\{z:z\in\mathbb{C} \text{ and } |z|<1\}.$$

In particular, for $n=1$, we write $\mathcal{A}_1=\mathcal{A}$.

For the functions $f(z)$ given by (1.1) and $g(z)$ given by

$$g(z)=z^p+\sum_{n=p+1}^{\infty}b_nz^n,$$

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their convolution (or Hadamard product) denoted by $f*g$, is defined by

$$(f*g)(z)=z^p+\sum_{n=p+1}^{\infty}a_nb_nz^n.$$

For two analytic functions f and g, the function f is subordinate to g, written as $f(z)\prec g(z)$ $(z\in\mathbb{U})$, if there exists a Schwarz function w, which (by definition) is analytic in $\mathbb{U}$ with $w(0)=0$ and $|w(z)|<1$ such that$f(z)=g(w(z))$ $(z\in\mathbb{U})$. It follows from this definition that

$$f(z)\prec g(z)\Longrightarrow f(0)=g(0) \quad \text{and} \quad f(\mathbb{U})\subset g(\mathbb{U}).$$

In particular, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence relation (see [9]).

$$f(z)\prec g(z)(z\in\mathbb{U})\Longleftrightarrow f(0)=g(0) \quad \text{and} \quad f(\mathbb{U})\subset g(\mathbb{U}).$$

Let $\phi(z)$ be analytic function in $\mathbb{U}$ with $\phi(0)=1$, $\phi'(0)>0$ and $\Re\{\phi(z)\}>0$ which maps the open unit disk $\mathbb{U}$ onto a region starlike with respect to 1 and is symmetric with respect to the real axis. In [1] Ali et al. defined and introduced the class $S^*_{b,p}(\phi)$ to be the class of function in $f\in\mathcal{A}_p$ for which

$$1+\frac{1}{b}\left(\frac{zf'(z)}{pf(z)}-1\right)\prec\phi(z), \quad (z\in\mathbb{U}, \quad b\in\mathbb{C}\setminus\{0\}),$$

and the corresponding class $C_{b,p}(\phi)$ of all functions in $f\in\mathcal{A}_p$ for which

$$1+\frac{1}{b}\left(\frac{1}{p}\left(1+\frac{zf''(z)}{f'(z)}\right)-1\right)\prec\phi(z), \quad (z\in\mathbb{U}, \quad b\in\mathbb{C}\setminus\{0\}).$$

Further, they also defined and studied the following classes:

$$R_{b,p}(\phi)=\left\{f\in\mathcal{A}_p:1+\frac{1}{b}\left(\frac{f'(z)}{pz^{p-1}}-1\right)\prec\phi(z),\ z\in\mathbb{U},\ b\in\mathbb{C}\setminus\{0\}\right\},$$
$$L^M_p(\alpha,\phi)=\left\{\frac{1-\alpha}{p}\frac{zf'(z)}{f(z)}+\frac{\alpha}{p}\left(1+\frac{zf''(z)}{f'(z)}\right)\prec\phi(z),\ z\in\mathbb{U},\ \alpha\geq 0\right\},$$

and

$$M_p(\alpha,\phi)=\left\{f\in\mathcal{A}_p:\frac{1}{p}\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}\prec\phi(z),\ z\in\mathbb{U},\ \alpha\geq 0\right\}.$$

Further, Ramachandran et al. [5] introduced the class $R_{p,b,\alpha,\beta}(\phi)$ to be the class of function in $f\in\mathcal{A}_p$ for which

$$1+\frac{1}{b}\left[(1-\beta)\left(\frac{f(z)}{z^p}\right)^{\alpha}+\beta\frac{zf'(z)}{pf(z)}\left(\frac{f(z)}{z^p}\right)^{\alpha}-1\right]\prec\phi(z), \quad (b\in\mathbb{C}\setminus\{0\},\ 0\leq\beta\leq 1,\ \alpha\geq 0).$$