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A Method for Solving Fredholm Integral Equations of the First Kind Based on Chebyshev Wavelets

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Abstract. In this paper, we suggest a method for solving Fredholm integral equation of the first kind based on wavelet basis. The continuous Legendre and Chebyshev wavelets of the first, second, third and fourth kind on [0,1] are used and are utilized as a basis in Galerkin method to approximate the solution of integral equations. Then, in some examples the mentioned wavelets are compared with each other.

Key Words: First kind Fredholm integral equation, Galerkin and Modified Galerkin method, Legendre wavelets, Chebyshev wavelets.

AMS Subject Classifications: 65R20, 65T60

1 Introduction

Many inverse problems in sciences and engineering lead to the solution of the following integral equation of the first kind,

$$\int_a^b k(x,t)y(t)dt = g(x), \quad a \le x \le b, \tag{1.1}$$

where $g(x)$ and $k(x,t)$ are known functions and $y(x)$ is the unknown function to be determined. In general, this kind of integral equation is ill-posed for a given kernel k and driving term g [6]. For this reason, the special method should be introduced to solve it. In recent years, several numerical methods for approximating the solution of Eq. (1.1) are known. Among them, the methods based on the wavelet are more attractive and considerable. The wavelet technique allows the creation of very fast algorithms when compared

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with known algorithms. Various wavelet bases are applied in order to solve Eq. (1.1). Legendre wavelets [11], Legendre multi wavelets [15], non-uniform Haar wavelets [10], wavelet moment method [5] wavelets-Galerkin method [12], CAS wavelets [7, 16] and Chebyshev wavelets of the first kind [2] have been used in order to approximate the solution of Eq. (1.1). In [4], by using the Chebyshev wavelets of the first kind, an integral equation of the second kind was solved based on the approximation of the kernel k and driving term g. In [2], by applying the Chebyshev wavelets of the first kind and the Galerkin method a numerical scheme was proposed for solving Eq. (1.1) based on the approximation of the kernel k and driving term g.

In this paper, we consider the Legendre, Chebyshev wavelets of the first kind (CFST), of the second kind (CSND), of the third kind (CTRD) and of the fourth kind (CFTH) [3, 4, 13] as basis functions in modified Galerkin method for numerical solution of Eq. (1.1) and compare them with each other. In Section 2, the Legendre and Chebyshev wavelets are reminded. The Galerkin method is presented in Section 3 and a comparison of these wavelets is proposed in Section 4 from the number of calculations. The proposed algorithm is introduced in Section 5 and tested in some numerical examples.

2 Preliminaries

Wavelets constitute a family of functions constructed from dilations and translations of a single function called mother wavelet. When the dilation parameter a and the translation parameter b vary continuously we have the following family of continuous wavelets as [1, 8],

$$\psi_{a,b}(t)=|a|^{-\frac{1}{2}}\psi\Big(\frac{t-b}{a}\Big), \quad a,b\in\mathbb{R}, \quad a\neq 0,$$

where $\psi_{a,b}(t)$ forms a wavelet basis for $L^2(\mathbb{R})$. We consider the parameters a and b as discrete values $a=a_0^{-k}$, $b=nb_0a_0^{-k}$, $a_0>1$, $b_0>0$, where n and k are positive integers. In particular, when $a_0=2$ and $b_0=1$, then $\psi_{k,n}(t)$ form an orthonormal basis.

Chebyshev wavelets $\psi_{n,m}=\psi(k,n,m,t)$ have four arguments, $n=1,2,\cdots,2^{k-1}$,k can assume any positive integer, m is the degree of Chebyshev polynomials and $t$denotes the time.

We denote $T_m(t)$, $U_m(t)$, $V_m(t)$ and $W_m(t)$ as the Chebyshev polynomials of the first, second, third and fourth kind of degree m which are defined respectively as follows [13, 14]:

$$T_m(t)=\cos m\theta, \qquad U_m(t)=\frac{\sin(m+1)\theta}{\sin\theta},$$

$$V_m(t)=\frac{\cos(m+\frac{1}{2})\theta}{\cos(\frac{1}{2}\theta)}, \qquad W_m(t)=\frac{\sin(m+\frac{1}{2})\theta}{\sin(\frac{1}{2}\theta)},$$

where $t=\cos\theta$ for $-1\le t\le 1$ and $0\le\theta\le\pi$ and they are orthogonal with respect to the weight functions $w(t)=1/\sqrt{1-t^2}$, $\sqrt{1-t^2}$, $\sqrt{1+t}/\sqrt{1-t}$, $\sqrt{1-t}/\sqrt{1+t}$ on the interval