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CERTAIN SUBCLASS OF $P-$ VALENT MEROMORPHIC ANALYTIC FUNCTIONS INVOLVING CERTAIN INTEGRAL OPERATOR

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Abstract. The purpose of the present paper is to introduce a new subclass of $p-$valent meromorphic functions by using certain integral operator and to investigate various properties for this subclass.

Key words: *analytic function, meromorphic function, hypergeometric function, linear operator*

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1 Introduction

Let $\sum_p$ denote the class of functions f of the form:

$$f(z)=\frac{1}{z^p}+\sum_{k=1-p}^{\infty}a_kz^k, \qquad p\in\mathbf{N}=\{1,2,3,\cdots\}, \tag{1.1}$$

which are analytic and $p-$valent in the punctured unit disc $U^*=\{z\in\mathbf{C}:0<|z|<1\}=U\setminus\{0\}$.

For a function f in the class $\sum_p$ given by (1.1), Aqlan et al.[1] introduced the following one parameter families of integral operator

$$\mathcal{P}_p^{\alpha}f(z)=\frac{1}{z^{p+1}\Gamma(\alpha)}\int_0^z\left(\log\frac{z}{t}\right)^{\alpha-1}t^{\alpha-1}f(t)\,\mathrm{d}t, \qquad \alpha>0; \quad p\in\mathbf{N} \tag{1.2}$$

Using an elementary integral calculus, it is easy to verify that

$$\mathcal{P}_p^{\alpha}f(z)=\frac{1}{z^p}+\sum_{k=1-p}^{\infty}\left(\frac{1}{k+p+1}\right)^{\alpha}a_kz^k, \qquad \alpha\geq 0; \quad p\in\mathbf{N}. \tag{1.3}$$

Also, it is easily verified from (1.3) that

$$z\left(\mathcal{P}_p^{\alpha}f(z)\right)'=\mathcal{P}_p^{\alpha-1}f(z)-(1+p)\,\mathcal{P}_p^{\alpha}f(z). \tag{1.4}$$

Definition. Let $\sum\limits_p^{\alpha}(\eta,\delta,\mu,\lambda)$ be the class of functions $f\in\sum\limits_p$ which satisfy the following inequality:

$$\Re\left\{(1-\lambda)\left(\frac{\mathcal{P}_p^{\alpha}f(z)}{\mathcal{P}_p^{\alpha}g(z)}\right)^{\mu}+\lambda\frac{\mathcal{P}_p^{\alpha-1}f(z)}{\mathcal{P}_p^{\alpha}g(z)}\left(\frac{\mathcal{P}_p^{\alpha}f(z)}{\mathcal{P}_p^{\alpha}g(z)}\right)^{\mu}\right\}>\eta, \tag{1.5}$$

where $g\in\sum_p$ satisfies the following inequality:

$$\Re\left\{\frac{\mathcal{P}_p^{\alpha}g(z)}{\mathcal{P}_p^{\alpha-1}g(z)}\right\}>\delta,\qquad 0\le\delta<1, z\in U, \tag{1.6}$$

and η,δ and μ are real numbers such that $0\le\eta,\delta<1$ and $\lambda\in\mathbf{C}$ with $\Re\{\lambda\}>0$.

To establish our main results we need the following lemmas.

Lemma 1[5]**.** *Let Ω be a set in the complex plane $\mathbf{C}$ and let the function $\psi:\mathbf{C}^2\to\mathbf{C}$ satisfy the condition $\psi(ir_2,s_1)\notin\Omega$ for all real $r_2,s_1\le-\frac{1+r_2^2}{2}$. If q is analytic in U with $q(0)=1$ and $\psi(q(z),zq'(z))\in\Omega, z\in U$, then*

$$\Re\{q(z)\}>0\ (z\in U).$$

Lemma 2[6]**.** *If q is analytic in U with $q(0)=1$, and if $\lambda\in C\backslash\{0\}$ with $\Re\{\lambda\}\geqslant 0$, then*

$$\Re\{q(z)+\lambda zq'(z)\}>\alpha,\qquad 0\le\alpha<1$$

implies

$$\Re\{q(z)\}>\alpha+(1-\alpha)(2\gamma-1),$$

where γ is given by

$$\gamma=\gamma(\Re\{\lambda\})=\int_0^1(1+t^{\Re\{\lambda\}})^{-1}\mathrm{d}t$$

which is increasing function of $\Re\{\lambda\}$ and $\frac{1}{2}\le\gamma<1$. The estimate is sharp in the sense that the bound cannot be improved.

For real or complex numbers a,b and $c\,(c\neq 0,-1,-2,\cdots)$, the Gaussian hypergeometric function is defined by

$${}_2F_1(a,b;c;z)=1+\frac{a.b}{c}\frac{z}{1!}+\frac{a(a+1).b(b+1)}{c(c+1)}\frac{z^2}{2!}+\cdots. \tag{1.7}$$

We note that the series (1.9) converges absolutely for $z\in U$ and hence represents an analytic function in U (see, for details, [7, Ch. 14]). Each of the identities (asserted by Lemma 3 below) is fairly well known (cf., e.g., [7, Ch. 14]).