Anal. Theory Appl.
Vol. 28, No. 3 (2012), 269–277
DOI : 10.3969/j.issn.1672-4070.2012.03.007

SOME NEW ITERATED FUNCTION SYSTEMS CONSISTING OF GENERALIZED CONTRACTIVE MAPPINGS

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Received May 22, 2012

Abstract. Iterated function systems (IFS) were introduced by Hutchinson in 1981 as a natural generalization of the well-known Banach contraction principle. In 2010, D. R. Sahu and A. Chakraborty introduced K-Iterated Function System using Kannan mapping which would cover a larger range of mappings. In this paper, following Hutchinson, D. R. Sahu and A. Chakraborty, we present some new iterated function systems by using the so-called generalized contractive mappings, which will also cover a large range of mappings. Our purpose is to prove the existence and uniqueness of attractors for such class of iterated function systems by virtue of a Banach-like fixed point theorem concerning generalized contractive mappings.

Key words: *iterated function system, attractor, generalized contractive mapping, complete metric space, fixed point*

AMS (2010) subject classification: 47H10, 54HA25

1 Introduction

As a natural generalization of the well-known Banach contraction principle, iterated function systems (IFS) were introduced by Hutchinson (see [8]) and popularized by Barnsley (see

Partially supported by National Natural Science Foundation of China (No. 10961003).

[4]). They represent one way of defining fractals as attractors of certain discrete dynamical systems. Moreover, they can be effectively applied to fractal image compressions. Therefore, it is no doubt that they have been attracting considerable attention of mathematicians and computer experts (see e.g. [3, 5-7, 10-13]). As observed recently in Andres[1], the same approach can be naturally extended to iterated multifunction systems (IMS) with resulting objects called multivalued fractals.

In this paper, the existence of new fractals is proved by means of the Banach-like theorem for so-called generalized contractive mappings. As a consequence, some new iterated function systems are founded, which are an important addition to Hutchinson's Iterated Function System and K-Iterated Function System.

2 Iterated Function Systems

In this section we recall some well known aspects of iterated function system used in the sequel (more complete and rigorous treatments may be found in [4] or [8]).

Let X denote a complete metric space with a distance function d and T be a mapping from X into itself. Then T is called a contraction mapping if there is a constant $0 \le s < 1$ such that

$$d(T(x), T(y)) \le sd(x, y).$$

Polish mathematician S. Banach proved a very important result, regarding contraction mapping in 1922, known as Banach Contraction Principle (see [2]).

Theorem 2.1[2]. *Let $T : X \to X$ be a contraction mapping on a complete metric space (X, d). Then T possesses exactly one fixed point $x^* \in X$. Moreover, for any point x in X, the sequence $\{T_n(x) : n = 0, 1, 2, \cdots\}$ converges to $x^* \in X$. That is $\lim_{n\to\infty} T_n(x) = x^*$ for each $x \in X$.*

In the famous paper [8], J.E. Hutchinson proved that, given a set of contractions in a complete metric space X, there exists a unique nonempty compact set $A \subset X$, named the attractor or fractal of the iterated functions system (IFS).

IFS generally employs contractive maps over a complete metric space (X, d), where the Banach's celebrated result mentioned above guarantees the existence and uniqueness of the fixed point known as "attractor" or "fractal". This can be done since the Hutchinson-Barnsley operator is also a contraction mapping over $H(X)$, where $H(X)$ denotes the space whose points are the compact subsets of X.

We now give some basic definitions and theorems concerning iterated function system, which are used in the proof below. Most of notations and results here is taken from [12].