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SEMI INHERITED BIVARIATE INTERPOLATION

Mohammad Ali Fariborzi Araghi and Amir Fallahzadeh
(*Islamic Azad University, Iran*)

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Abstract. The bivariate interpolation in two dimensional space $\mathbf{R}^2$ is more complicated than that in one dimensional space $\mathbf{R}$, because there is no Haar space of continuous functions in $\mathbf{R}^2$. Therefore, the bivariate interpolation has not a unique solution for a set of arbitrary distinct pairwise points. In this work, we suggest a type of basis which depends on the points such that the bivariate interpolation has the unique solution for any set of distinct pairwise points. In this case, the matrix of bivariate interpolation has the semi inherited factorization.

Key words: *inherited factorization, inherited interpolation, semi inherited interpolation, bivariate interpolation, interpolation matrix*

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1 Introduction

In recent years, the bivariate and multivariate interpolations have been studied in the papers [3,8,9,12,13,16]. In [6,7] the inherited interpolation of matrices has been introduced by using the LU inherited factorization of a matrix. In this paper, we develop the idea to offer a type of bivariate interpolation which is based on the semi inherited LU factorization of the interpolation matrix. The factorization of matrices is a method to solve the square system of linear equations. One kind of these factorizations is LU factorization which has various types. One of them is the inherited LU factorization which has been described in [1] by M. Arav and et al. In this work, we use a special kind of the inherited LU factorization. We call this special type of LU factorization as semi inherited LU factorization. Then, we constitute the bivariate interpolation matrix such that it has the semi inherited LU factorization. At first, in section 2, we introduce the semi inherited LU factorization[1,10] and present some preliminaries of interpolation briefly[14,15]. In section 3, we describe the semi inherited bivariate interpolation which depends on the semi inherited LU factorization and we prove that the interpolation matrix in this case has the semi inherited LU factorization. In section 4, we illustrate the mentioned method by some numerical examples.

2 Preliminaries

Definition 2.1. Let A be an $n\times n$ matrix that $a_{ii}\neq 0$ for $i=1,\cdots,n$ and write $A=B+D+C$ that B is strictly lower triangular, D is diagonal, and C is strictly upper triangular. Then, A has the semi inherited LU factorization if and if only $A=(I+BD^{-1})(D+C)$. (I is the identity matrix.)

According to the definition 2.1, the following theorem is proved.

Theorem 2.2. *Let $A=B+D+C$ be a decomposition of the matrix A with invertible diagonal entries where B is strictly lower triangular, D is diagonal and C is strictly upper triangular. Then, A has the semi inherited LU factorization if and only if $BD^{-1}C=0$.*

Proof. Suppose the matrix $A_{n\times n}$ has the semi inherited LU factorization. Then,

$$A=B+D+C=(I+BD^{-1})(D+C)=D+C+B+BD^{-1}C\Rightarrow BD^{-1}C=0.$$

Conversely, suppose $BD^{-1}C=0$. Then,

$$A=B+D+C+BD^{-1}C\Rightarrow A=(I+BD^{-1})(D+C).$$

For example,

$$\begin{aligned} A &= \begin{pmatrix} 2 & 0 & 12 & 6\\ 0 & 6 & 12 & 6\\ 2 & -6 & 1 & 0\\ -4 & 12 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 1 & -1 & 1 & 0\\ -2 & 2 & -3 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 12 & 6\\ 0 & 6 & 12 & 6\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}\\ &= (I+BD^{-1})(D+C). \end{aligned}$$

Remark 2.3. A useful property of matrices that have semi inherited LU factorization is this fact that the calculation of LU factorization of them is very easy, because the matrix U is inherited entirely and the matrix L is the product of a strictly lower triangular matrix in the diagonal matrix plus the identity matrix[1].

Suppose $X=\{x_1,x_2,\dots,x_n\}$ is a set of n distinct points in $\mathbf{R}^n$ $(n\geq 1)$. We call X the node set. Also, for each x_i the value $z_i\in\mathbf{R}$ is given. The problem of interpolation is to find a suitable function $F:\mathbf{R}^n\longrightarrow\mathbf{R}$ such that $F(x_i)=z_i$ for $i=1,\cdots,n$. F is called the interpolation function of $\{(x_i,z_i)\}_{i=1}^n$.

Let U be a vector space of functions with the basis $\{u_1,u_2,\cdots,u_n\}$. We can consider F in the form of $F=\sum\limits_{j=1}^{n}\lambda_j u_j$. Then,

$$F(x_i)=z_i,\quad \forall i=1,\cdots,n \quad\text{implies}\quad \sum_{j=1}^{n}\lambda_j u_j(x_i)=z_i,\quad \forall i=1,\cdots,n\ .$$

i.e.,

$$\begin{pmatrix} u_1(x_1) & u_2(x_1) & \cdots & u_n(x_1)\\ u_1(x_2) & u_2(x_2) & \cdots & u_n(x_2)\\ \vdots & \vdots & & \vdots\\ u_1(x_n) & u_2(x_n) & \cdots & u_n(x_n) \end{pmatrix} \begin{pmatrix} \lambda_1\\ \lambda_2\\ \vdots\\ \lambda_n \end{pmatrix} = \begin{pmatrix} z_1\\ z_2\\ \vdots\\ z_n \end{pmatrix}.$$