Anal. Theory Appl.
Vol. 27, No. 1 (2011), 76–91
DOI10.1007/s10496-011-0076-9

APPROXIMATING COMMON FIXED POINTS OF NEARLY ASYMPTOTICALLY NONEXPANSIVE MAPPINGS

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Received Oct. 9, 2010

Abstract. We use an iteration scheme to approximate common fixed points of nearly asymptotically nonexpansive mappings. We generalize corresponding theorems of [1] to the case of two nearly asymptotically nonexpansive mappings and those of [9] not only to a larger class of mappings but also with better rate of convergence.

Key words: *iteration scheme, nearly asymptotically nonexpansive mapping, rate of convergence, common fixed point, weak and strong convergence*

AMS (2010) subject classification: 47H09, 47H10

1 Introduction

Throughout this paper, $\mathbf{N}$ denotes the set of all positive integers. Let E be a real Banach space and C a nonempty subset of E. A mapping $T : C \to C$ is called asymptotically nonexpansive if for a sequence $\{k_n\} \subset [1,\infty)$ with $\lim\limits_{n\to\infty} k_n = 1$, we have

$$\|T^n x - T^n y\| \le k_n \|x-y\|$$

for all $x, y \in C$ and $n \in \mathbf{N}$. T is called uniformly L-Lipschitzian if for some $L > 0$, $\|T^n x - T^n y\| \le L\|x-y\|$ for all $x, y \in C$ and $n \in \mathbf{N}$. Also, T is called a contraction if for some $0 < k < 1$, $\|Tx - Ty\| \le k\|x-y\|$ for all $x, y \in C$.

Fix a sequence $\{a_n\} \subset [0,\infty)$ with $\lim\limits_{n\to\infty} a_n = 0$, then according to Agarwal et al[1], T is said to be nearly asymptotically nonexpansive if $k_n \ge 1$ for all $n \in \mathbf{N}$ with $\lim\limits_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \le k_n(\|x-y\| + a_n)$$

for all $x, y \in C$. T will be nearly uniformly L-Lipschitzian if $k_n \leq L$ for all $n \in \mathbf{N}$.

Note that every asymptotically nonexpansive mapping is nearly asymptotically nonexpansive and every nearly asymptotically nonexpansive mapping is nearly uniformly L-Lipschitzian.

We know that Picard and Mann iteration processes for a mapping $T : C \to C$ are defined as:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = Tx_n\,,\ n \in \mathbf{N} \end{cases} \tag{1.1}$$

and

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n,\ \ n \in \mathbf{N} \end{cases} \tag{1.2}$$

respectively, where $\{\alpha_n\}$ is in $(0,1)$.

Recently, Agarwal et al.[1] introduced the following iteration scheme:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-\alpha_n)T^n x_n + \alpha_n T^n y_n, \\ y_n = (1-\beta_n)\,x_n + \beta_n T^n x_n,\ \ n \in \mathbf{N}, \end{cases} \tag{1.3}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in $(0,1)$. They showed that this scheme converges at a rate same as that of Picard iteration.

On the other hand, we state without error terms the iteration scheme studied by Yao and Chen [9] for common fixed points of two mappings:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = \alpha_n x_n + \beta_n T^n x_n + \gamma_n S^n x_n,\ n \in \mathbf{N}, \end{cases} \tag{1.4}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in $[0,1]$ and $\alpha_n + \beta_n + \gamma_n = 1$. They did not show the rate of convergence of this scheme.

We introduce the following iteration scheme to compute the common fixed points of two mappings.

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1-\alpha_n)T^n x_n + \alpha_n S^n y_n, \\ y_n = (1-\beta_n)\,x_n + \beta_n T^n x_n,\, n \in \mathbf{N}, \end{cases} \tag{1.5}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in $(0,1)$.

It is to be noted that (1.5) reduces to