Anal. Theory Appl.
Vol. 27, No. 1 (2011), 59–75
DOI10.1007/s10496-011-0059-x

BOUNDEDNESS OF PARABOLIC SINGULAR INTEGRALS AND MARCINKIEWICZ INTEGRALS ON TRIEBEL-LIZORKIN SPACES

Yaoming Niu
(*Baotou teachers College, China*)
Shuangping Tao
(*Northwest Normal University, China*)

Received Mar 29, 2010

Abstract. In this paper, we obtain the boundedness of the parabolic singular integral operator T with kernel in $L(\log L)^{1/\gamma}(S^{n-1})$ on Triebel-Lizorkin spaces. Moreover, we prove the boundedness of a class of Marcinkiewicz integrals $\mu_{\Omega,q}(f)$ from $\|f\|_{\dot{F}_p^{0,q}(\mathbf{R}^n)}$ into $L^p(\mathbf{R}^n)$.

Key words: *parabolic singular integral, Triebel-Lizorkin space, Marcinkiewica integral, rough kernel*

AMS (2010) subject classification: 42B25, 42B35

1 Introduction

Let S^{n-1} denote the unit sphere on the n-dimension Euclidean space $\mathbf{R}^n$ and $\beta_n \geq \beta_{n-1} \geq \cdots \geq \beta_1 \geq 1$ be fixed real numbers. For each fixed $x = (x_1, \cdots, x_n) \in \mathbf{R}^n$, the function

$$F(x,\rho) = \sum_{i=1}^{n} \frac{x_i^2}{\rho^{2\beta_i}}$$

is strictly decreasing of $\rho > 0$. Therefore, there exists a unique $\rho = \rho(x)$ such that $F(x,\rho) = 1$. Define $\rho(x) = t$ and $\rho(0) = 0$. It is proved in [10] that ρ is a metric on $\mathbf{R}^n$ and $(\mathbf{R}^n, \rho)$ is called the mixed homogeneity space related to $\{\beta_i\}_{i=1}^n$. For any $x = (x_1, x_2, \cdots, x_n) \in \mathbf{R}^n$, let

$$x_1 = \rho^{\beta_1} \cos\varphi_1 \ldots \cos\varphi_{n-2} \cos\varphi_{n-1},$$

Supported in part by National Natural Foundation of China (Grant No. 11071250).

$$x_2 = \rho^{\beta_2}\cos\varphi_1 \dots \cos\varphi_{n-2}\sin\varphi_{n-1},$$
$$\dots\dots\dots\dots\dots\dots\dots$$
$$x_{n-1} = \rho^{\beta_n - 1}\cos\varphi_1\sin\varphi_2,$$
$$x_n = \rho^{\beta_n}\sin\varphi_1.$$

Then $dx = \rho^{\beta-1}J(x')d\rho d\sigma$, where $\beta = \sum_{i=1}^{n}\beta_i, x' \in S^{n-1}$, and $\rho^{\beta-1}J(x')$ is the Jacobian of the above transform. In [10] Fabes and Rivière pointed out that $J(x')$ is a C^∞ function on S^{n-1}, and $1 \le J(x') \le M$. For $\lambda > 0$, let $B_\lambda = \mathrm{diag}[\lambda^{\beta_1}, \cdots, \lambda^{\beta_n}]$ be a diagonal matrix. We say a real valued measurable function $\Omega(x)$ is homogeneous of degree zero with respect to B_λ if for any $\lambda > 0$ and $x \in \mathbf{R}^n$

$$\Omega(B_\lambda x) = \Omega(x). \tag{1.1}$$

Moreover, we assume that $\Omega(x)$ satisfies the condition

$$\int_{S^{n-1}} \Omega(x')J(x')\mathrm{d}\sigma(x') = 0. \tag{1.2}$$

Let $\alpha > 0$ and

$$L(\log L)^\alpha(S^{n-1}) = \left\{\Omega : \int_{S^{n-1}} |\Omega(y')|\log^\alpha(2+|\Omega(y')|)\mathrm{d}\sigma(y') < \infty\right\}.$$

It is well known that the following relations hold:

$$L^q(S^{n-1})(q>1) \subseteq L\log^+ L(S^{n-1}) \subseteq H^1(S^{n-1}) \subseteq L^1(S^{n-1}),$$

$$L(\log L)^\beta(S^{n-1}) \subseteq L(\log L)^\alpha(S^{n-1}), 0 < \alpha < \beta,$$

$$L(\log L)^\alpha(S^{n-1}) \subseteq H^1(S^{n-1}), \ \ \alpha \ge 1,$$

where $H^1(S^{n-1})$ is the Hardy space on the unit sphere. While

$$L(\log L)^\alpha(S^{n-1}) \nsubseteq H^1(S^{n-1}) \nsubseteq L(\log L)^\alpha(S^{n-1}), \qquad 0 < \alpha < 1.$$

For $\gamma \ge 1$, let $\Delta_\gamma(\mathbf{R}^+)$ be the set of all measurable functions h on $\mathbf{R}^+$ satisfying the condition

$$\sup_{R>0}\left(R^{-1}\int_0^R |h(t)|^\gamma \mathrm{d}t\right)^{1/\gamma} < \infty,$$

and $\Delta_\infty(\mathbf{R}^+) = L^\infty(\mathbf{R}^+)$. Also, define $H_\gamma(\mathbf{R}^+)$ to be the set of all measurable functions h on $\mathbf{R}^+$ satisfying the condition

$$\|h\|_{L^\gamma(\mathbf{R}^+, \frac{\mathrm{d}t}{t})} = \left(\int_{\mathbf{R}^+} |h(t)|^\gamma \frac{\mathrm{d}t}{t}\right)^{1/\gamma} \le 1,$$