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A UNIFIED THREE POINT APPROXIMATING SUBDIVISION SCHEME

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Abstract. In this paper, we propose a three point approximating subdivision scheme, with three shape parameters, that unifies three different existing three point approximating schemes. Some sufficient conditions for subdivision curve C^0 to C^3 continuity and convergence of the scheme for generating tensor product surfaces for certain ranges of parameters by using Laurent polynomial method are discussed. The systems of curve and surface design based on our scheme have been developed successfully in garment CAD especially for clothes modelling.

Key words: *approximating subdivision scheme, shape parameters, Laurent polynomial*

AMS (2010) subject classification: 65D17, 65D07, 65D05

1 Introduction

In recent years, the subdivision scheme became one of the most popular methods of creating geometric object in computer aided geometric design and in animation industry. Their popularity is due to the facts that subdivision algorithms are easy to implement and suitable for computer applications. If the limit curve / surface approximates the initial control polygon and that after subdivision, the newly generated control points are not in the limit curve / surface, the scheme is said to be approximating. It is called interpolating if after subdivision, the control points of the original control polygon and the new generated control points are interpolated on the limit curve / surface. The important schemes for applications should allow to control the shape of the limit curve and be capable of reproducing families of curves widely used in computer graphics. A wide variety of schemes that has been proposed in the literature which posses shape parameters [2,4,5,8,9,15] are interpolating[3,11,16], presented approximating subdivision schemes with tension

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parameters. Zhijie Cai[17] developed the systems of curve and surface design for garment CAD by taking nonuniform control points. Hassan et al[6] proposed approximating three point binary scheme which has C^3 continuity. Kai Hormann et al[10] offered the dual three-point scheme which has quadratic precision. Recently, Siddiqi et al [2] presented a new three point approximating C^2 subdivision scheme. The smoothness of subdivision schemes using Laurents polynomial method has been discussed by [1,5,13] and [6]. The aim of this work is to offer an approximating three point scheme that captures Hassan et al[6], Kai Hormann et al[10] and Siddiqi et al[12] schemes and offer more flexibility in curve and surface drawing. Here the proposed scheme is C^3 as well as accompanied with three shape parameters, which helps in designing more than a single parameter. It also provides more flexibility in designing, especially in cloth modeling.

2 Preliminaries

A general form of univariate subdivision scheme S which maps a polygon $f^k = \{f_i^k\}_{i\in\mathbf{Z}}$ to a refined polygon $f^{k+1} = \{f_i^{k+1}\}_{i\in\mathbf{Z}}$ is defined by

$$\begin{cases} f_{2i}^{k+1} = \sum\limits_{j\in\mathbf{Z}} a_{2j} f_{i-j}^k, \\ f_{2i+1}^{k+1} = \sum\limits_{j\in\mathbf{Z}} a_{2j+1} f_{i-j}^k, \end{cases} \tag{2.1}$$

where the set $a = \{a_i | i \in \mathbf{Z}\}$ of coefficients is called mask of the subdivision scheme. A necessary condition for the uniform convergence of the subdivision scheme (2.1) is that

$$\sum_{j\in\mathbf{Z}} a_{2j} = \sum_{j\in\mathbf{Z}} a_{2j+1} = 1. \tag{2.2}$$

For analysis of the subdivision scheme with mask a, it is very practical to consider the z-transform of the mask,

$$a(z) = \sum_{i\in\mathbf{Z}} a_i z^i, \tag{2.3}$$

which is usually called the symbol/Laurent polynomial of the scheme. From (2.2) and (2.3) the Laurent polynomial of a convergent subdivision scheme satisfies

$$a(-1) = 0 \quad \text{and} \quad a(1) = 2. \tag{2.4}$$

This condition guarantees the existence of a related subdivision scheme for the divided differences of the original control points and the existence of associated Laurent polynomial $a^{(1)}(z)$ which can be defined as follows:

$$a^{(1)}(z) = \frac{2z}{1+z} a(z).$$